Continuous-variable quantum information, multi-mode quantum optics and bosonic error correcting codes

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Stiftung / Foundation

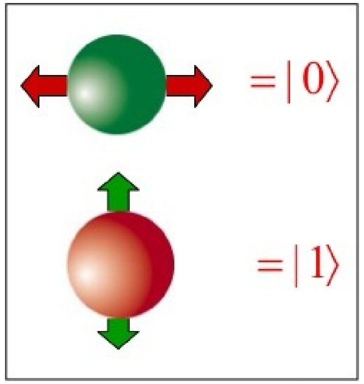
Freie Universität Berlin

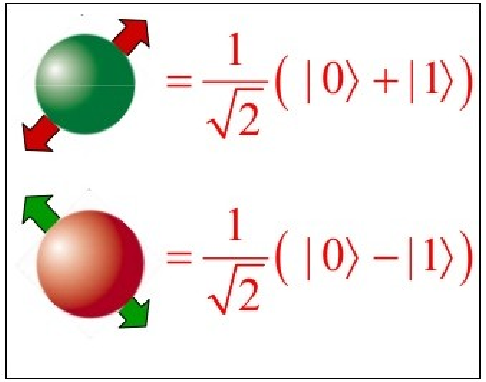
Discrete and continuous variables

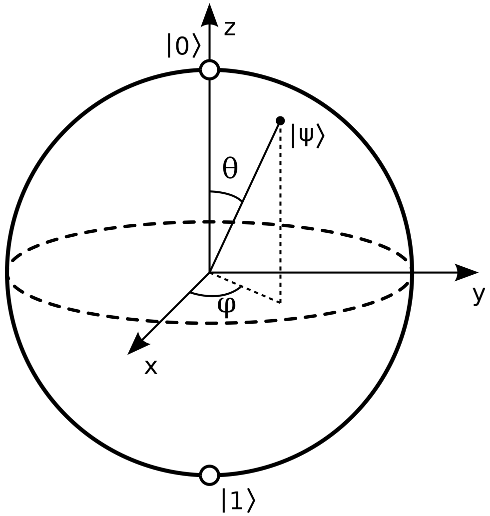
DV :

information encoded in *d*-level systems
(typically *d = 2*, ***qubits***)

$$\alpha|0\rangle + \beta|1\rangle$$







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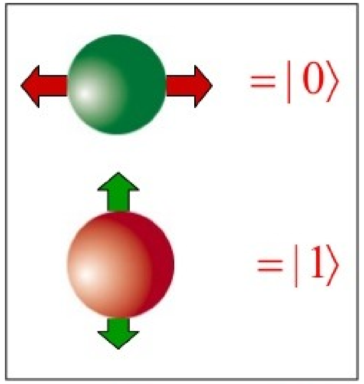

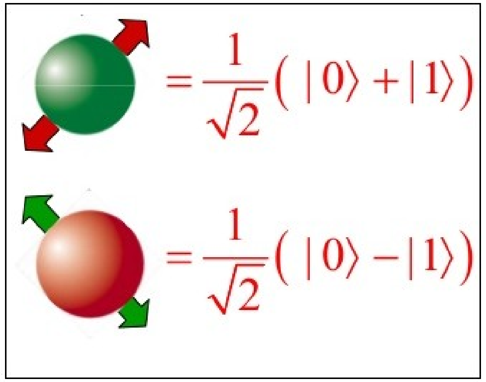

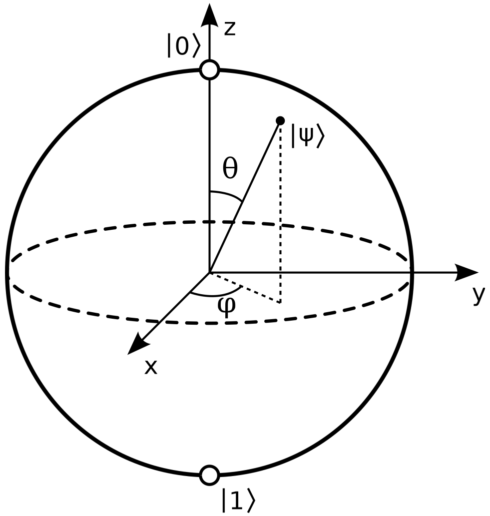

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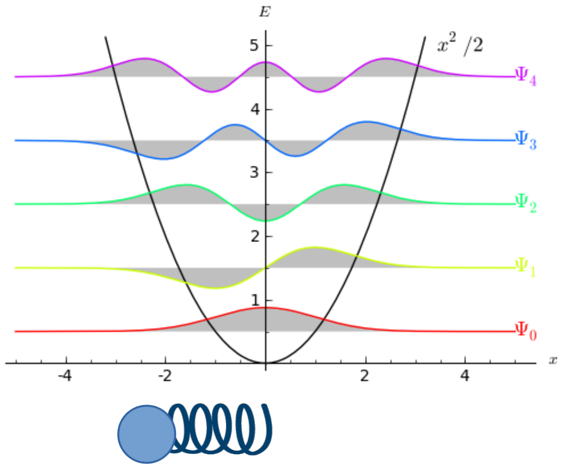
e.g. : $\hat{q}$, $\hat{p}$



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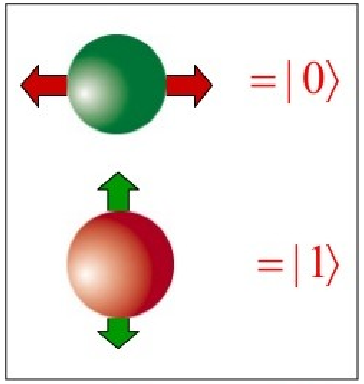

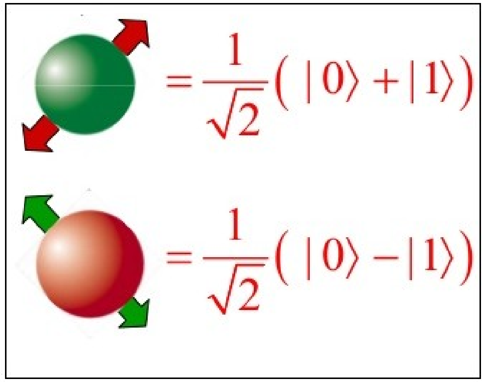

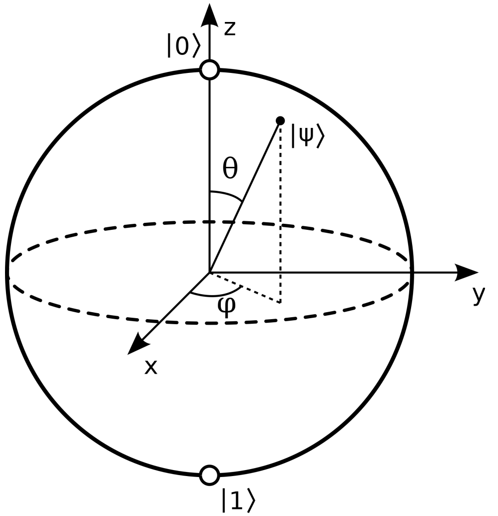

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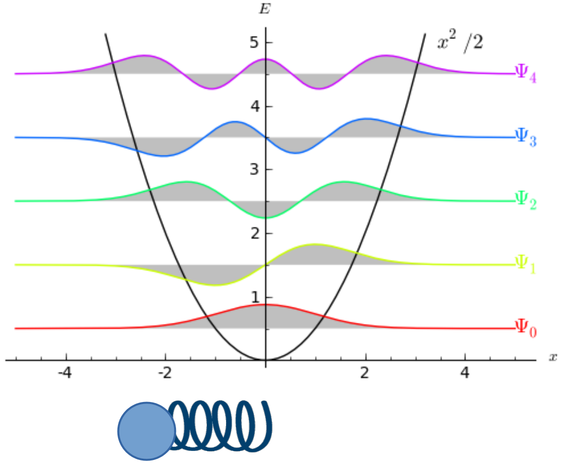

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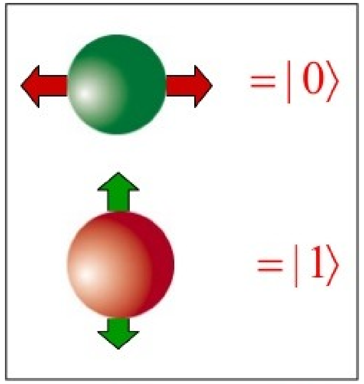

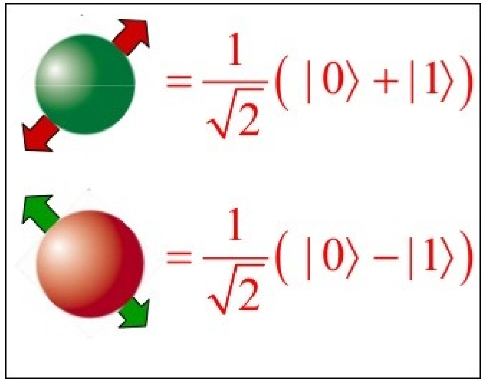

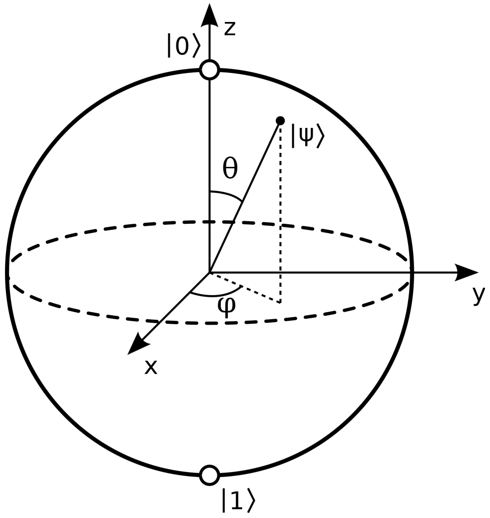

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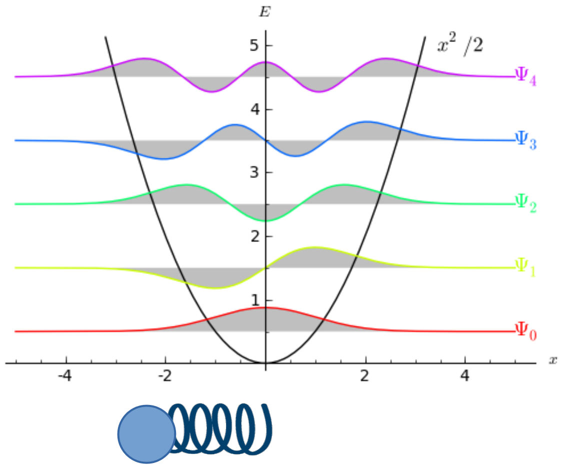
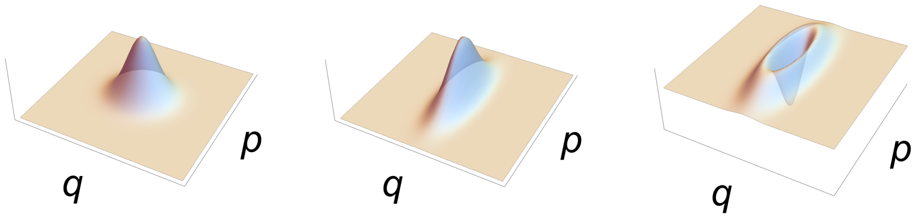


In phase space:
Wigner Function



Quasi-probability distribution

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Why optics?

Why CV?

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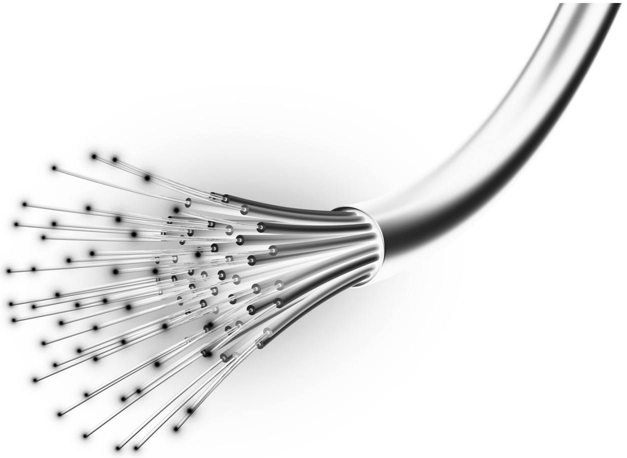

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⇨ Lots of know-how, technology

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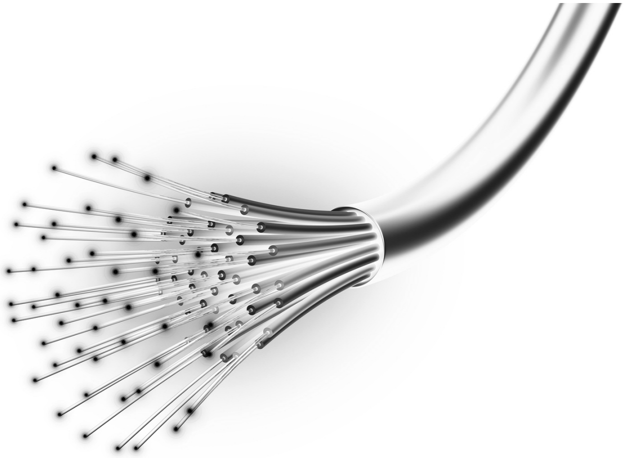

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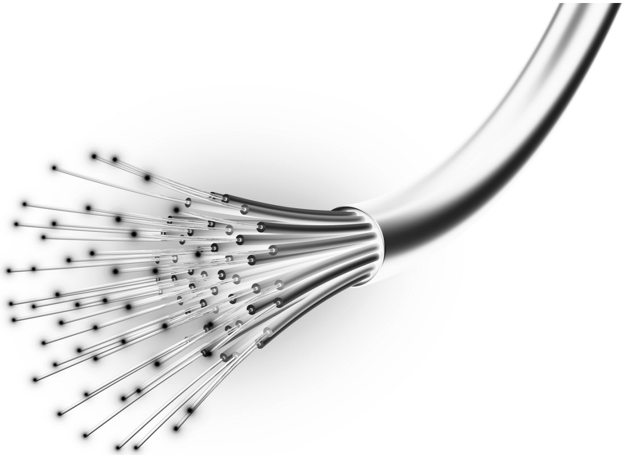

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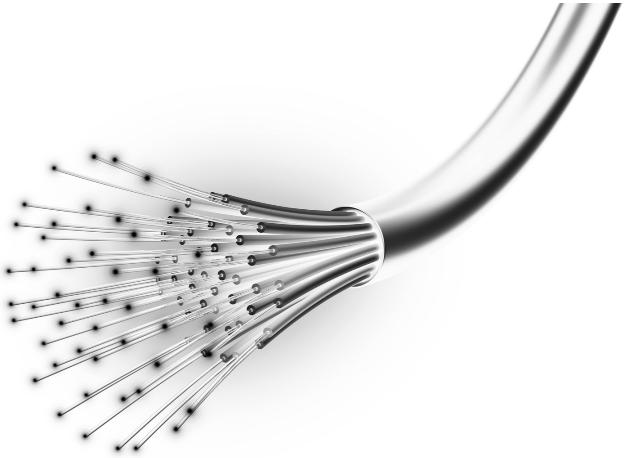

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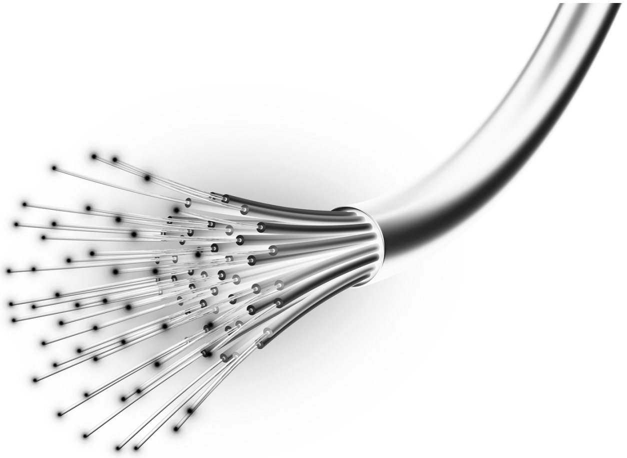

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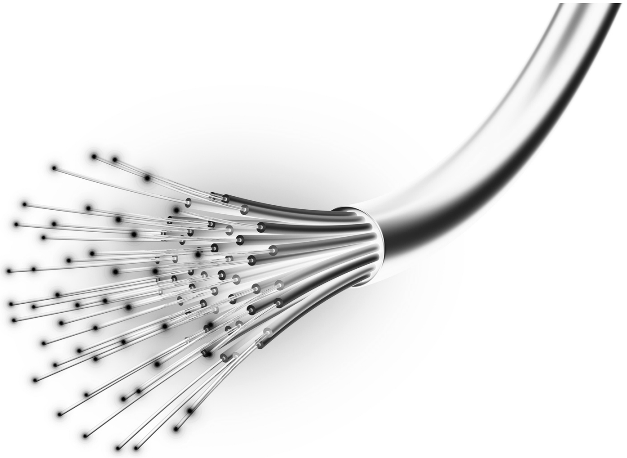

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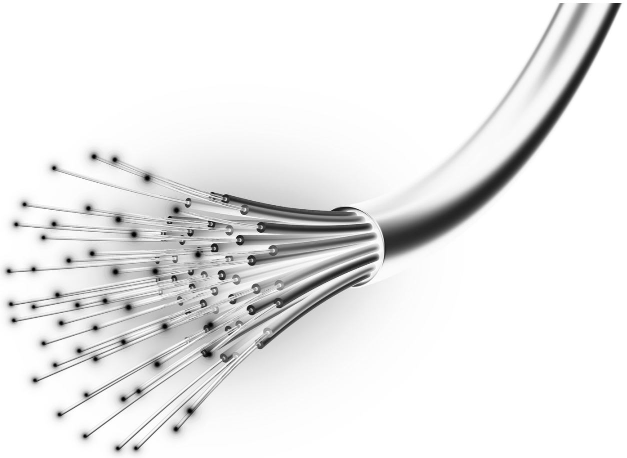

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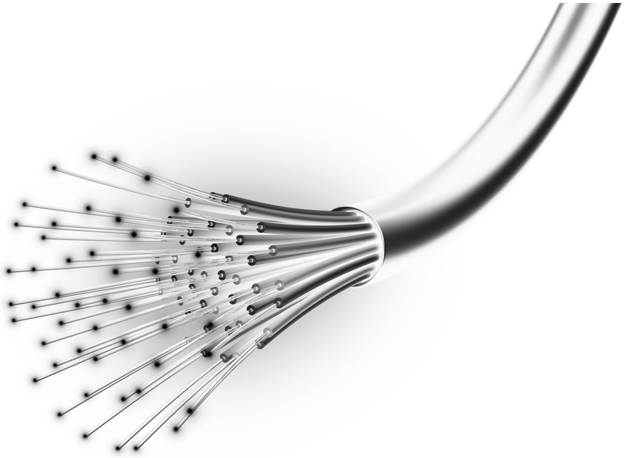

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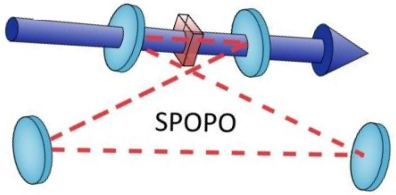
- Loss-resistant (ECC codes)
- Complementary “easy” operations with respect to other systems (hybrid devices)
- New sets of problems (Boson sampling)

Past research

Overview



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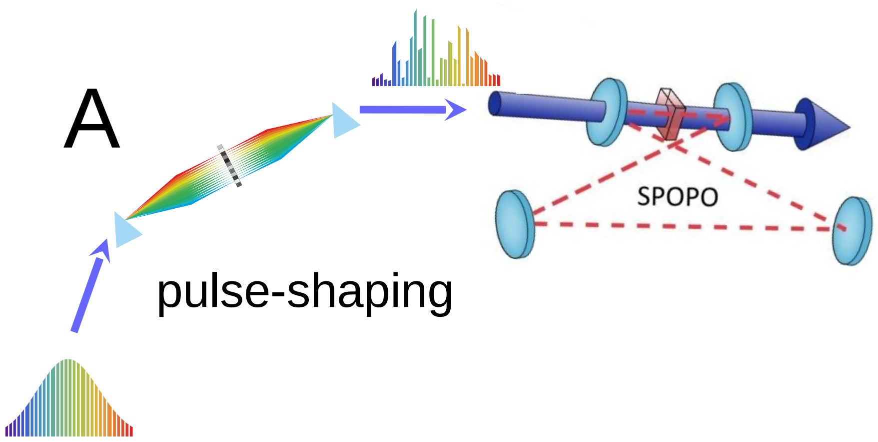

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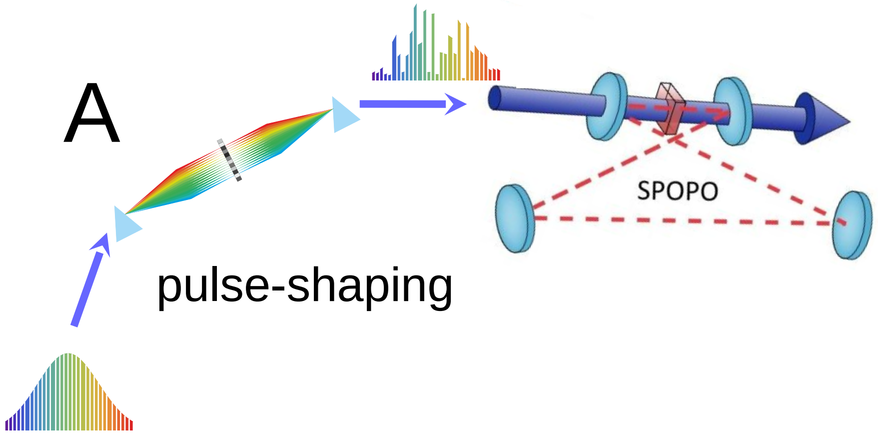
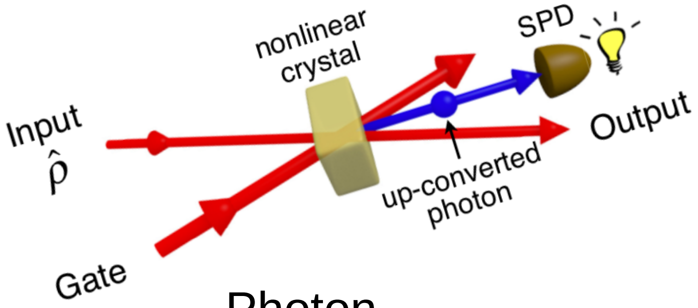


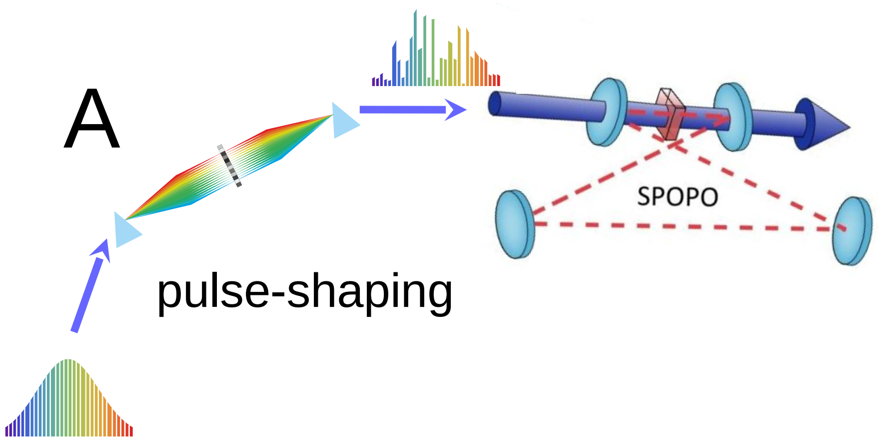
pulse-shaping

B



Photon subtraction

Overview



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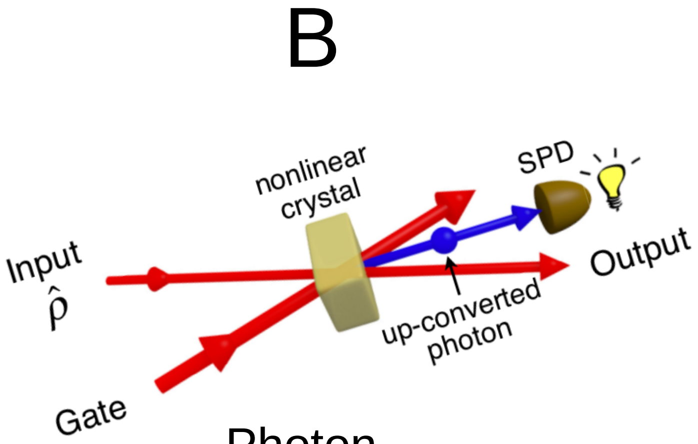

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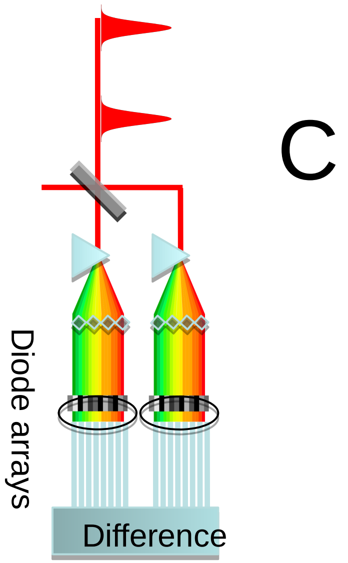

Multi-pixel
(or shaped) HD

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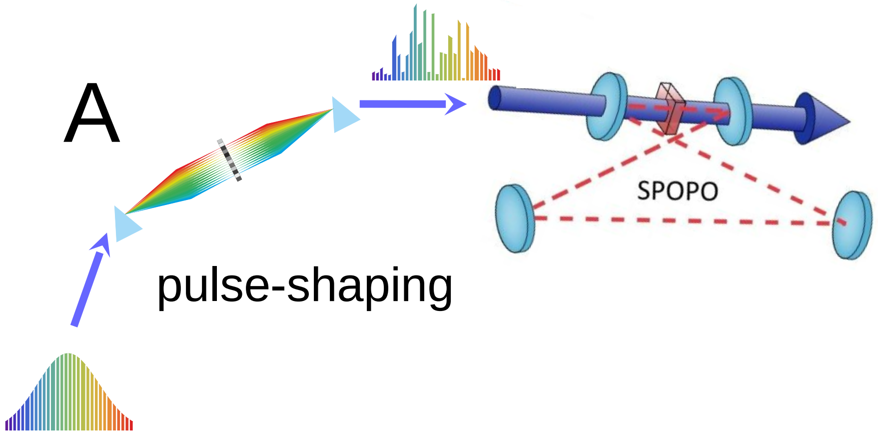

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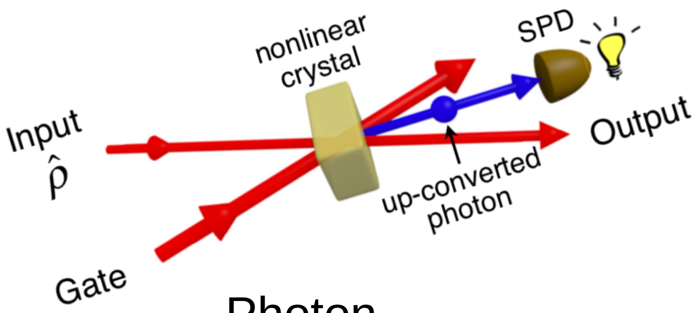
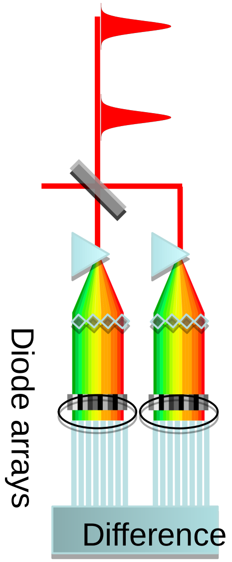


Photon
subtraction

C



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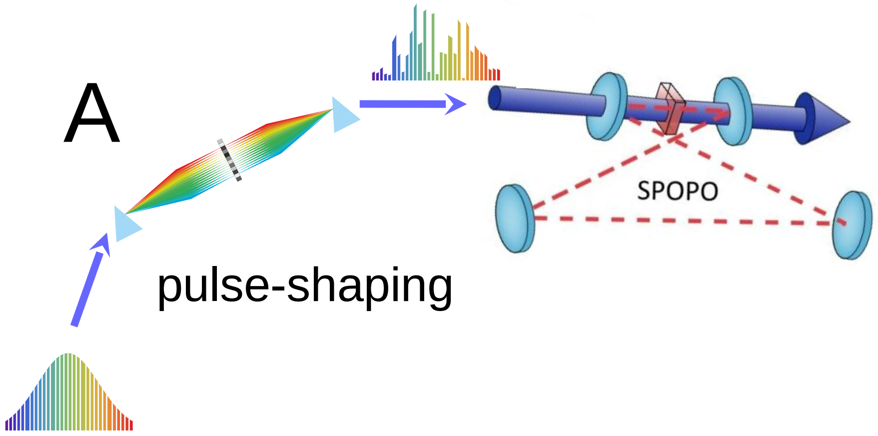

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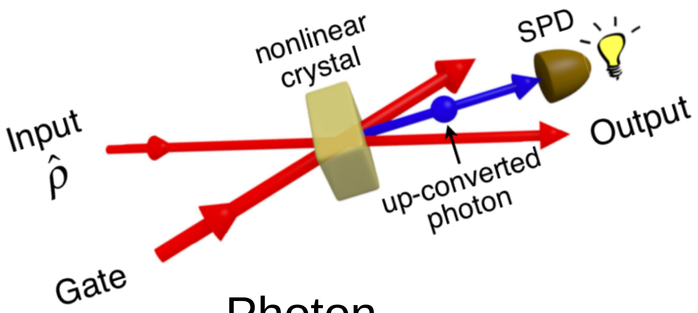

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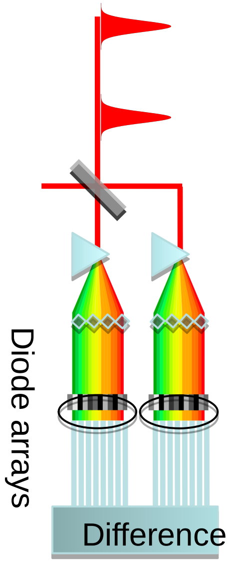

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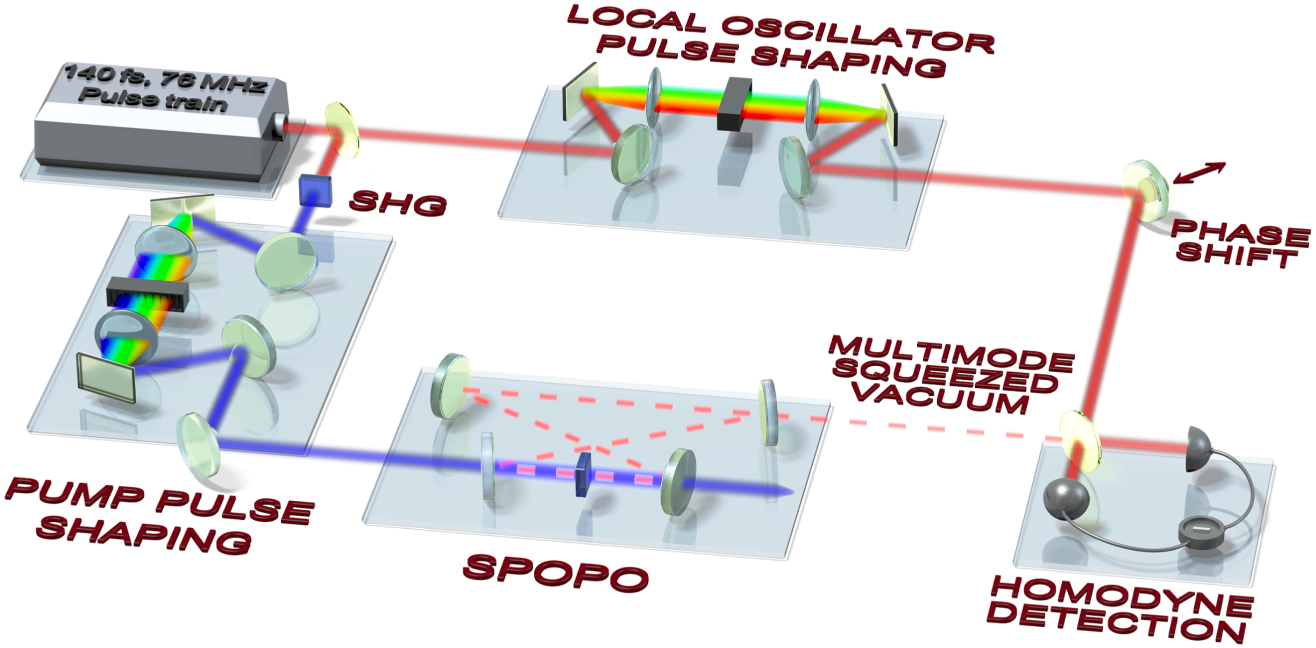
Image by
Valérian Thiel

Motivation: Experimental setup can generate multi-mode, entangled states of light.
Such states can be resources for quantum information processing.



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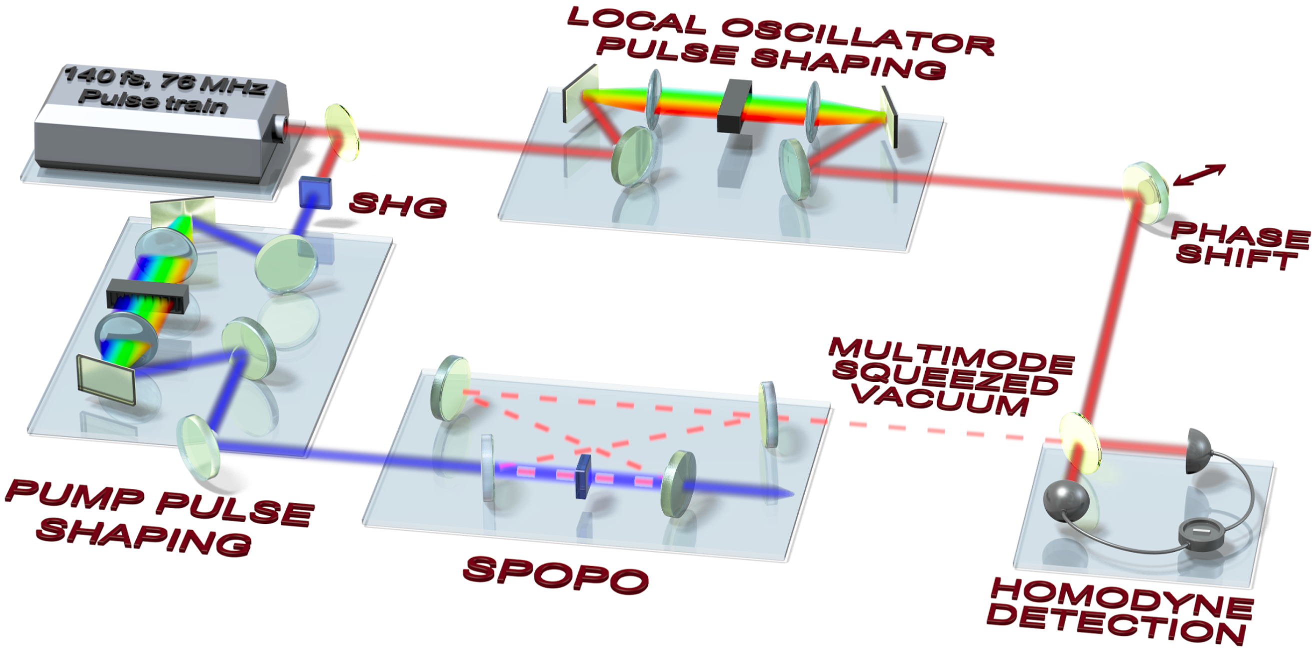

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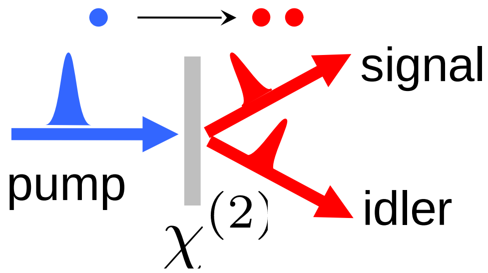

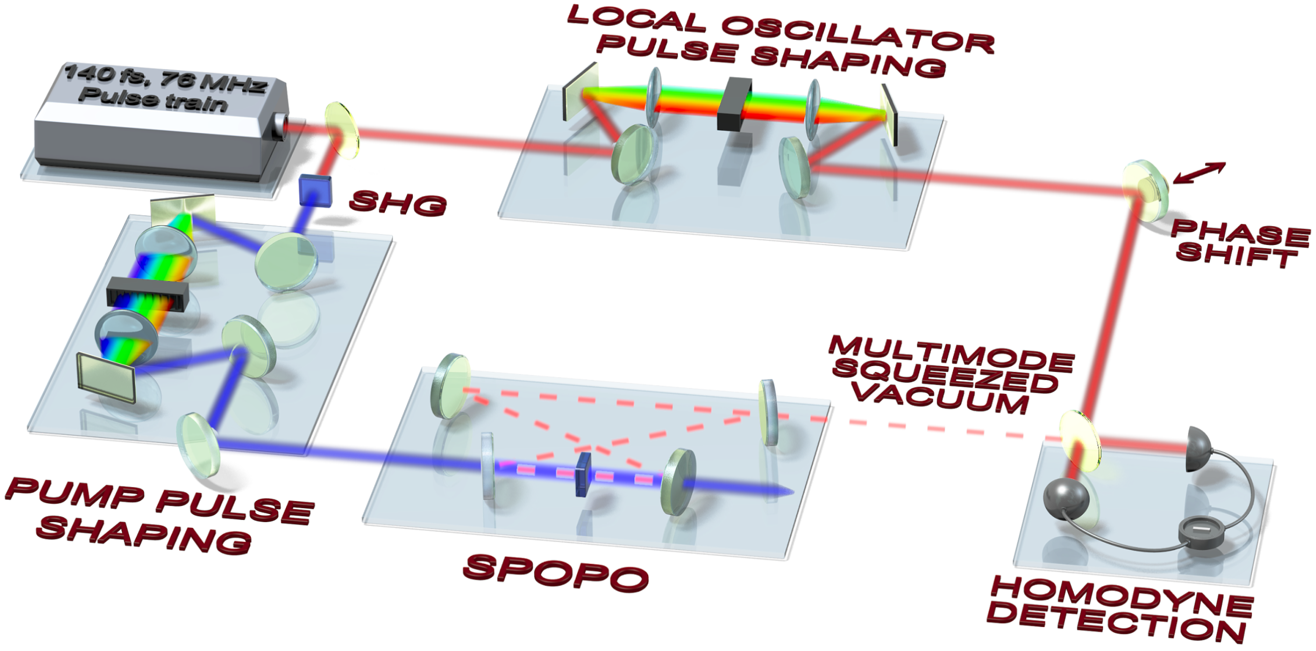

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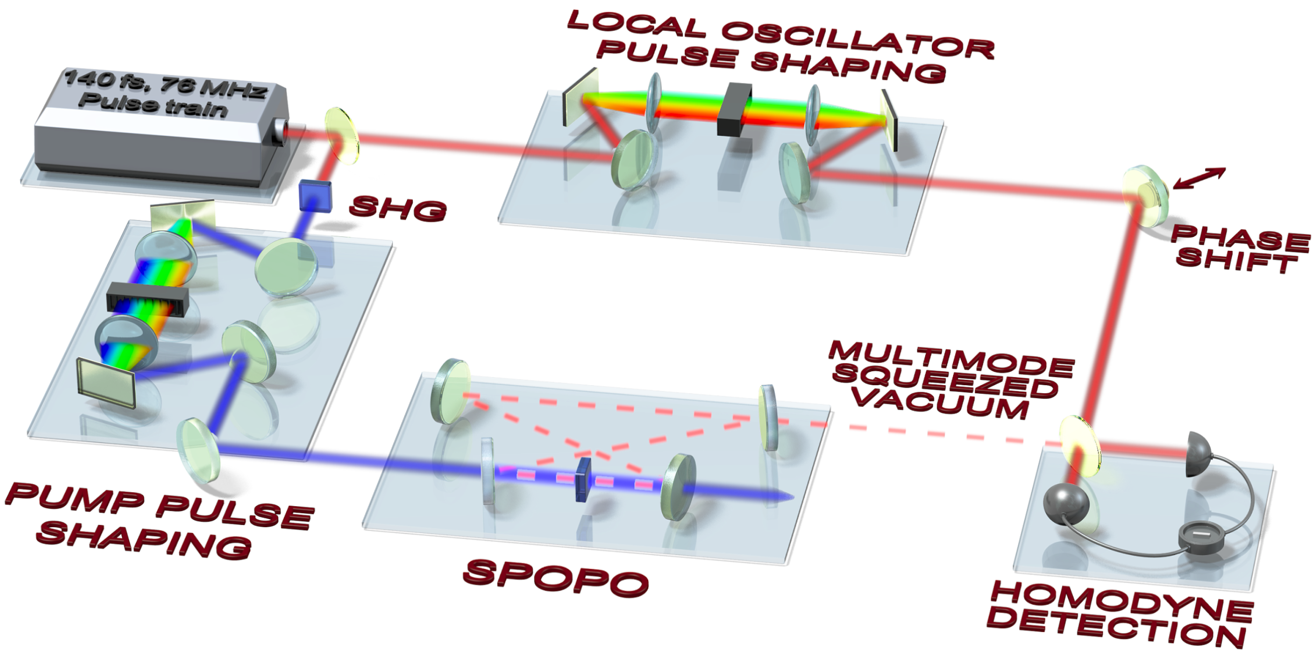
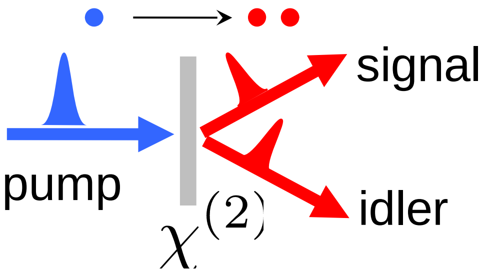
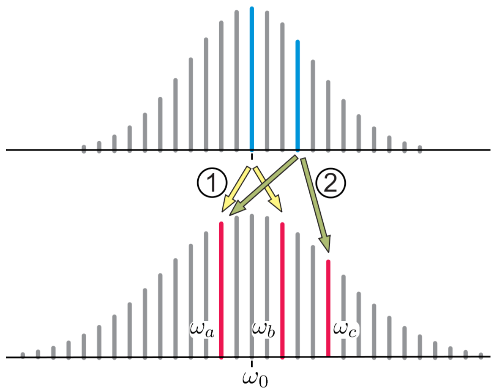




Entanglement from spontaneous parametric down-conversion of optical frequency combs



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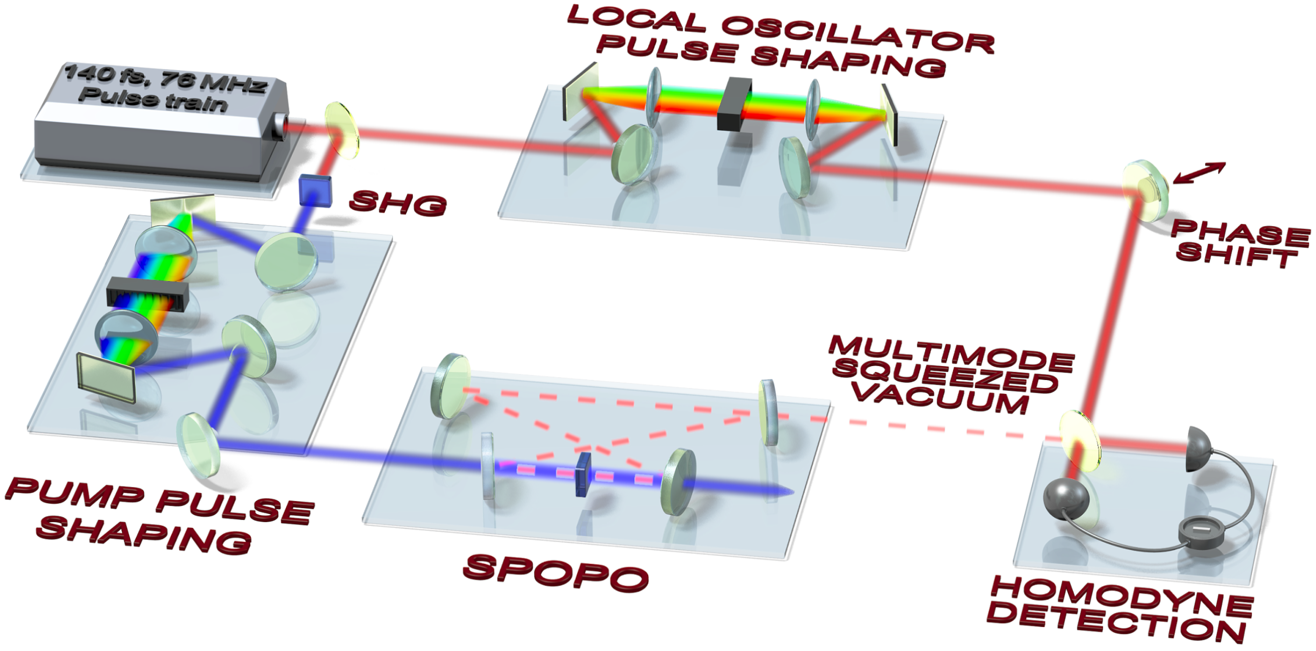

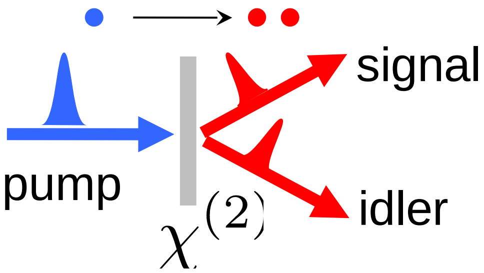

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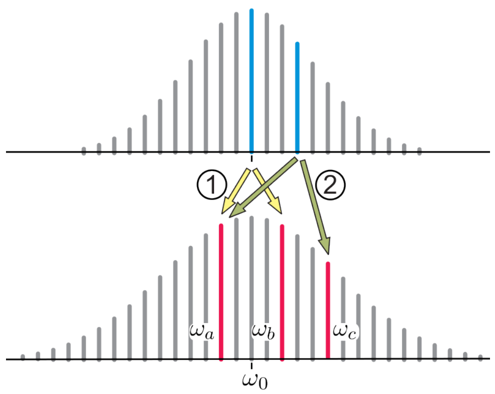

Effective quadratic Hamiltonian:

$$H = i \sum_{m,q} \mathcal{L}_{m,q} \hat{a}^{\dagger}_{\omega_m} \hat{a}^{\dagger}_{\omega_q} + \mathrm{h.c.}$$

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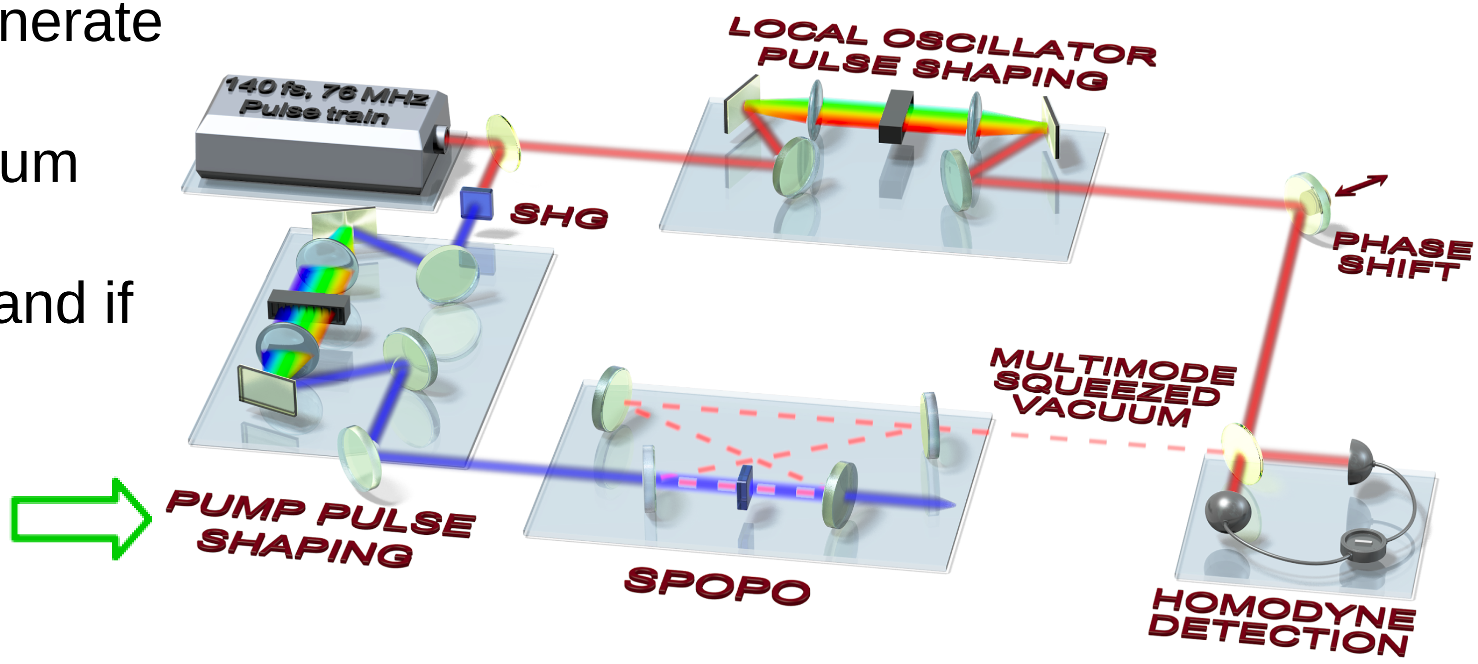

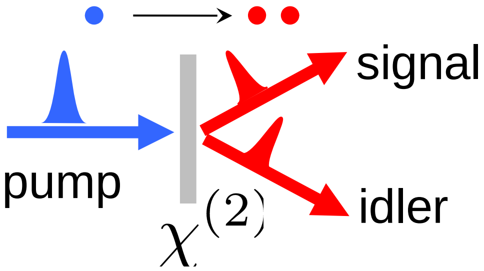

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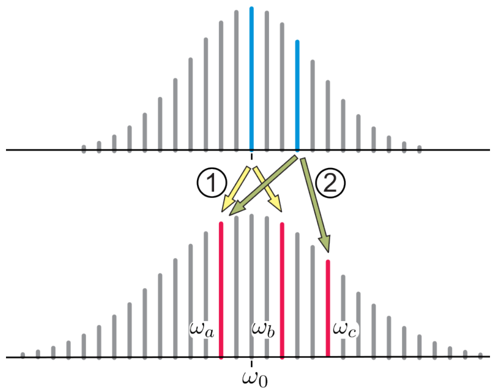

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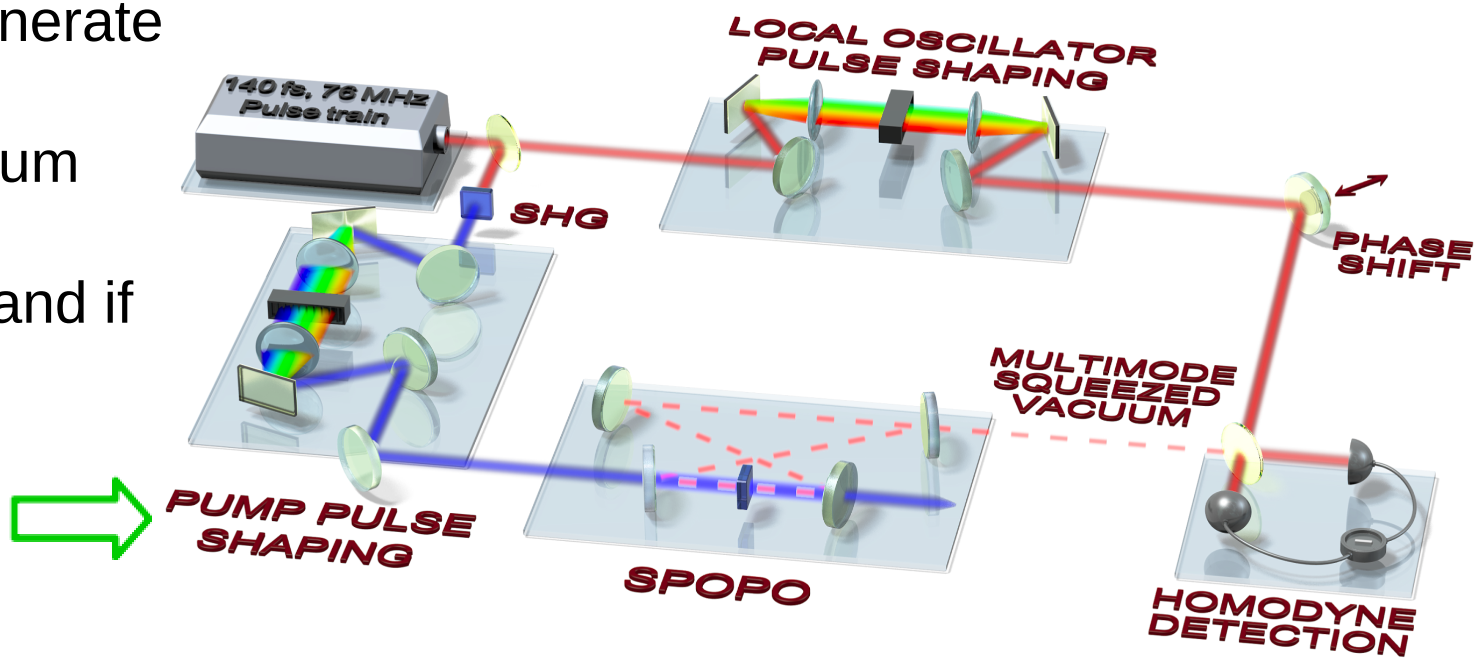

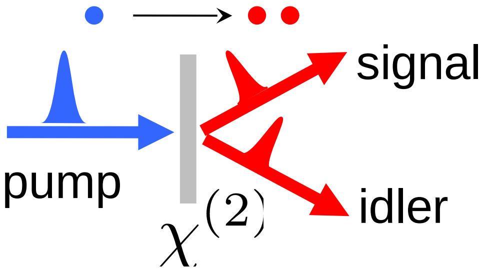

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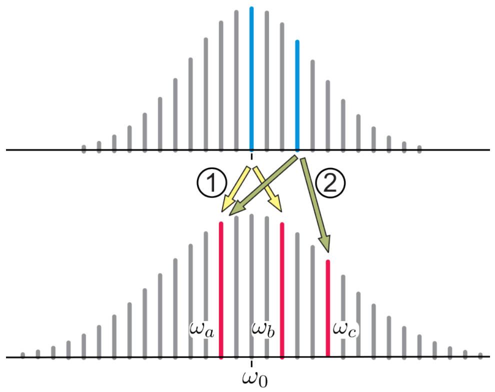

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Our contribution:

Combining pump-shape and detection system optimization it is possible to realize QIP including

- Measurement-based Q comp.
- Simulation of complex networks (Turku)
- Secret sharing (more later)

Non-Gaussianity from single-photon detection

Motivation:

1. No Q Advantage without non-Gaussian
2. Realizable non-Gauss: single photon ops

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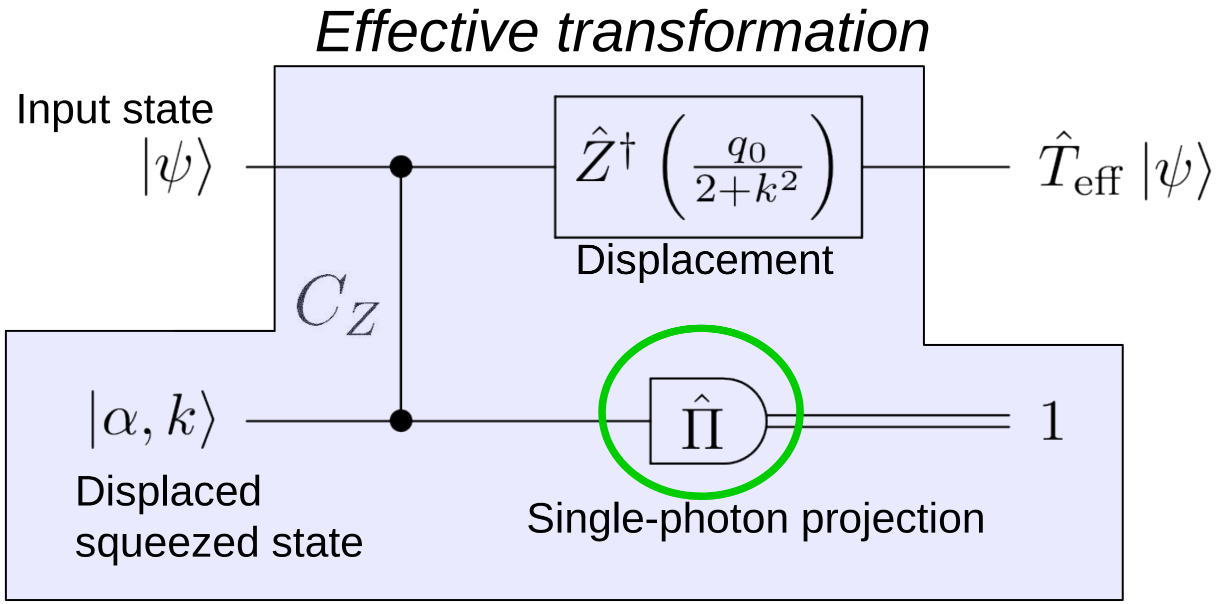

Effective transformation

$$\hat{T}_{\text{eff}} = \tilde{\mathcal{N}} \exp\left\{-\left(\frac{k^2}{4+2k^2}\right)\left(\hat{q}+p_0\right)^2\right\}\left(\hat{q}-\lambda\left(\alpha, k\right)\right)$$

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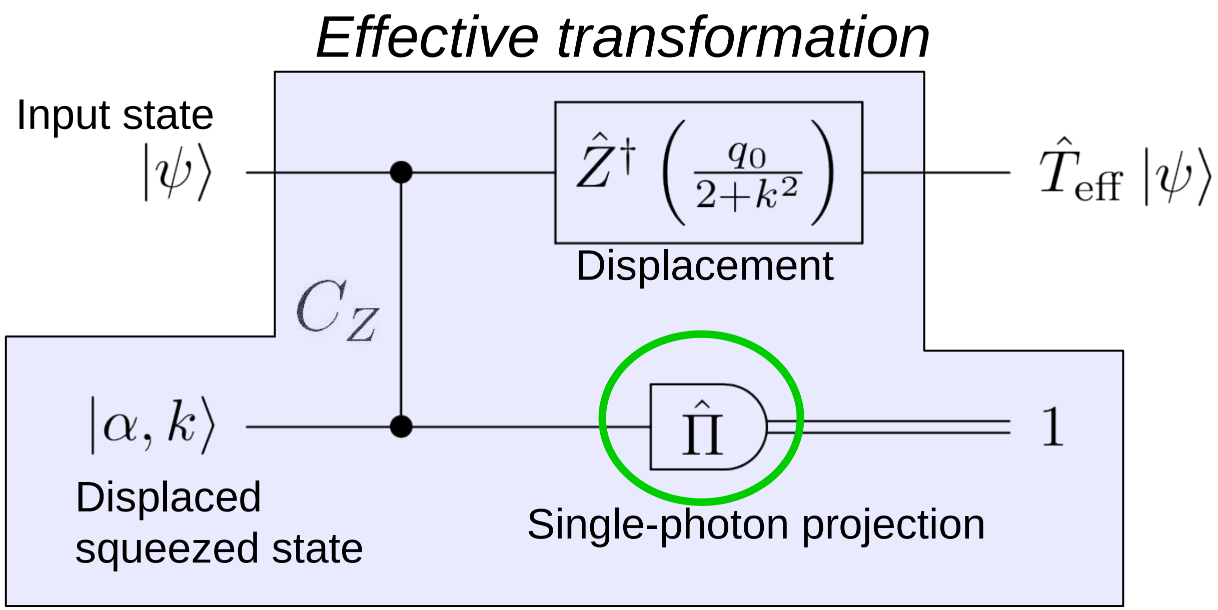

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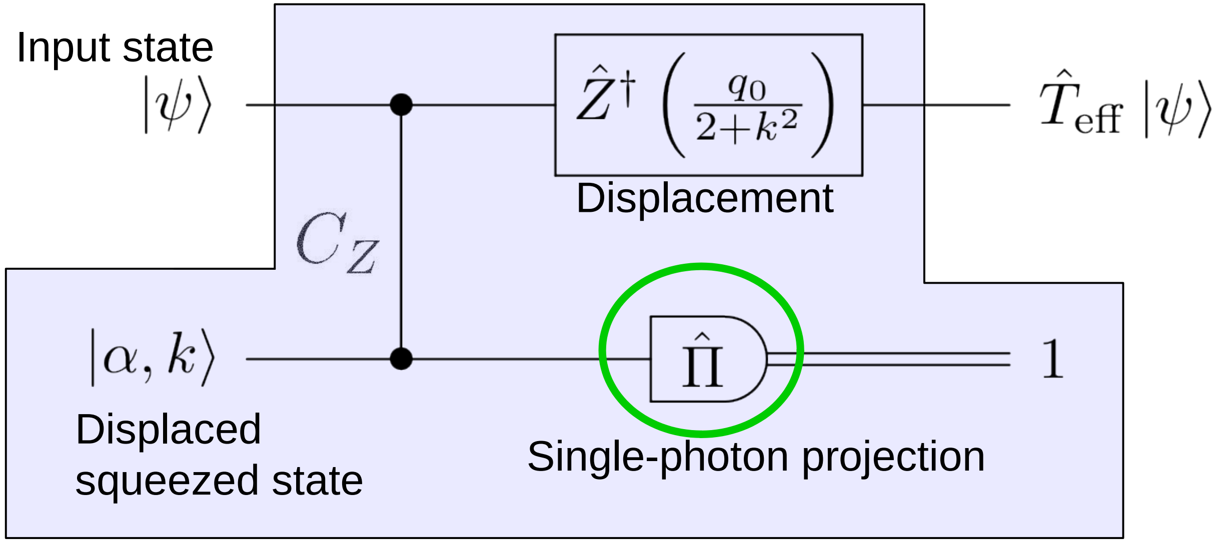

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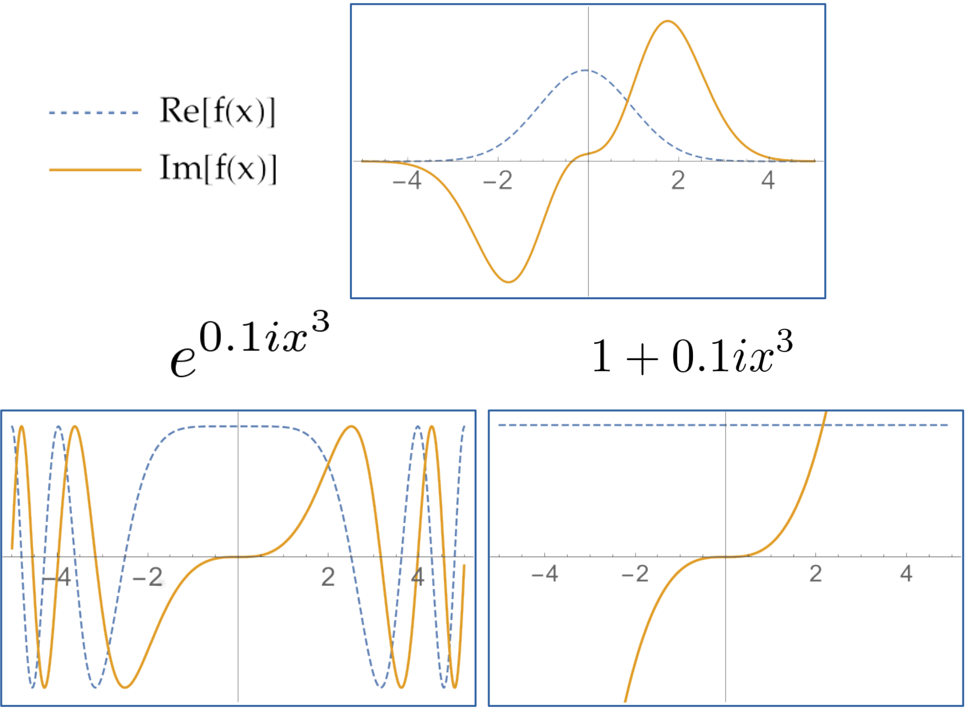
$f(\hat{q}) \to f(x)$



$$e^{i\nu\hat{q}^3} \approx \mathbb{I} + i\nu\hat{q}^3 = (\hat{q}-\lambda_1)(\hat{q}-\lambda_2)(\hat{q}-\lambda_3)$$

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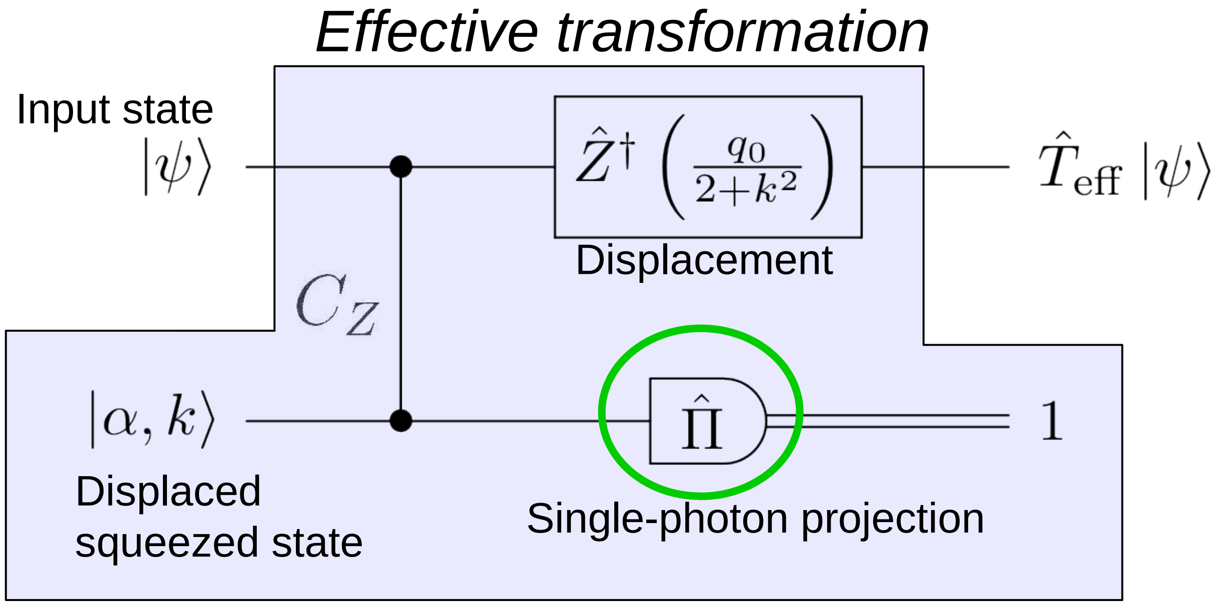

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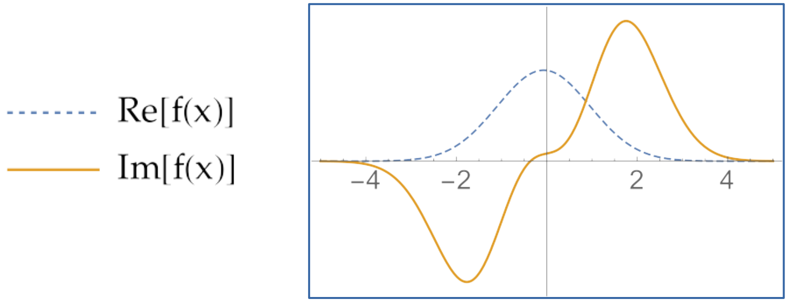

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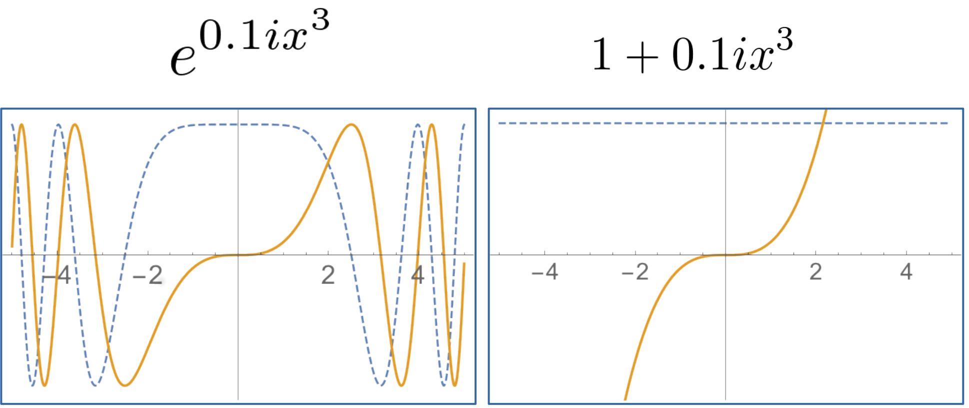

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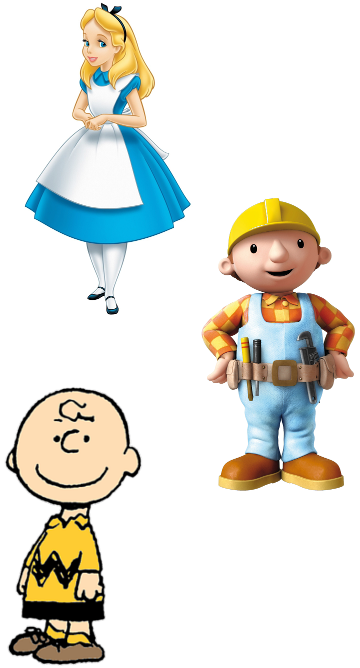
<u>Our contribution:</u>

Single-photon non-unitary operations can be used to approximate non-Gaussian unitary evolution

CV quantum state sharing: random codes

A **dealer** shares a **secret** with several **players** such that only **authorized subsets** of players can retrieve it if they **collaborate**

s



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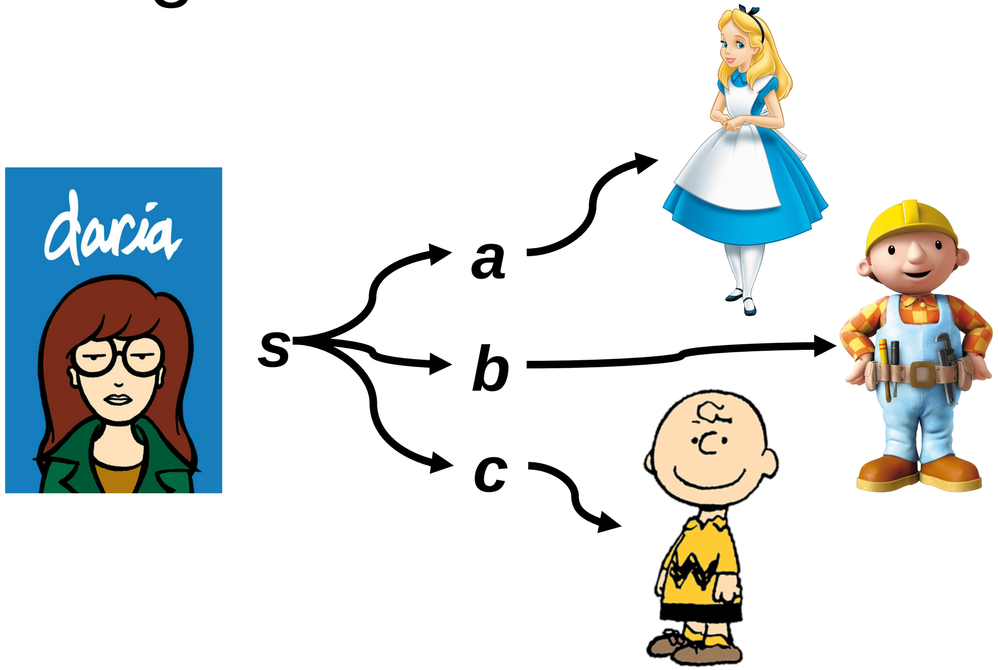

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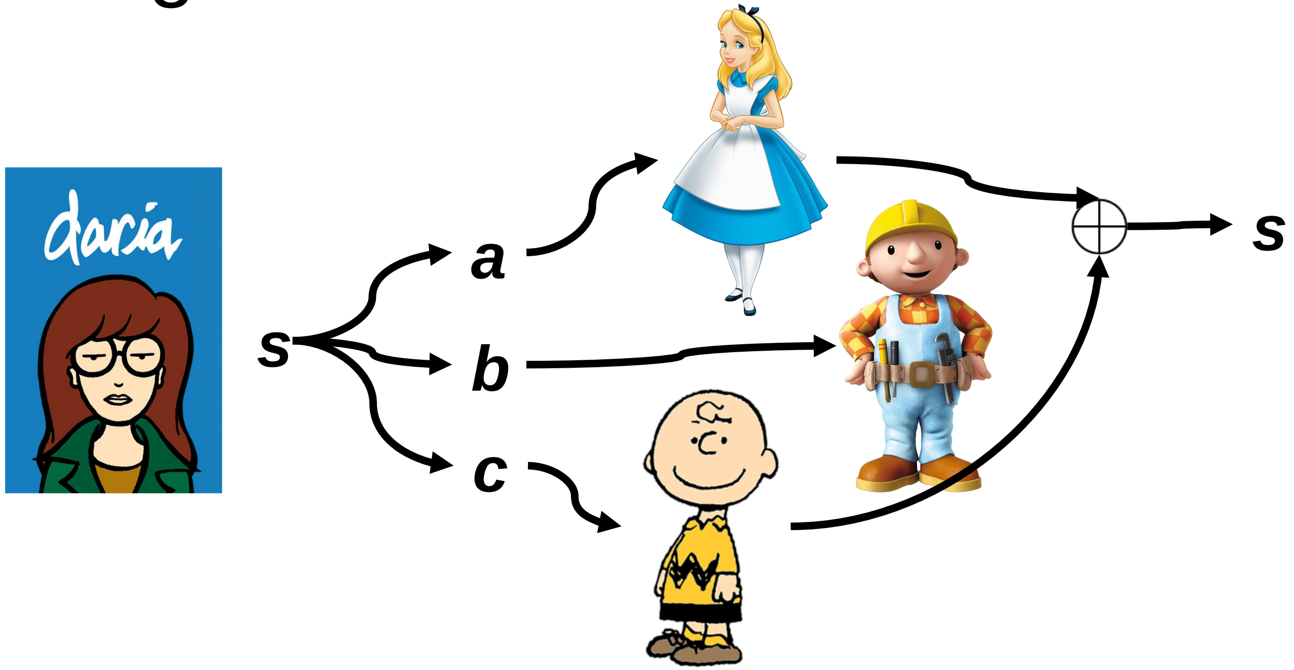

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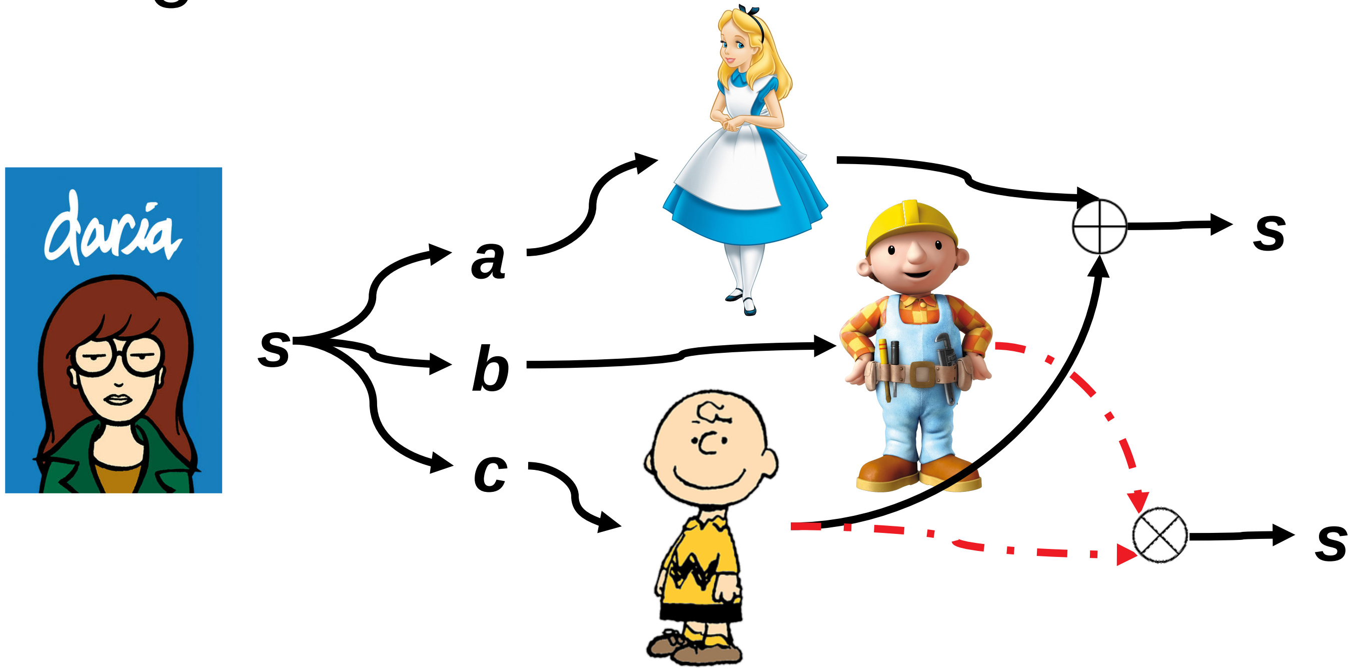

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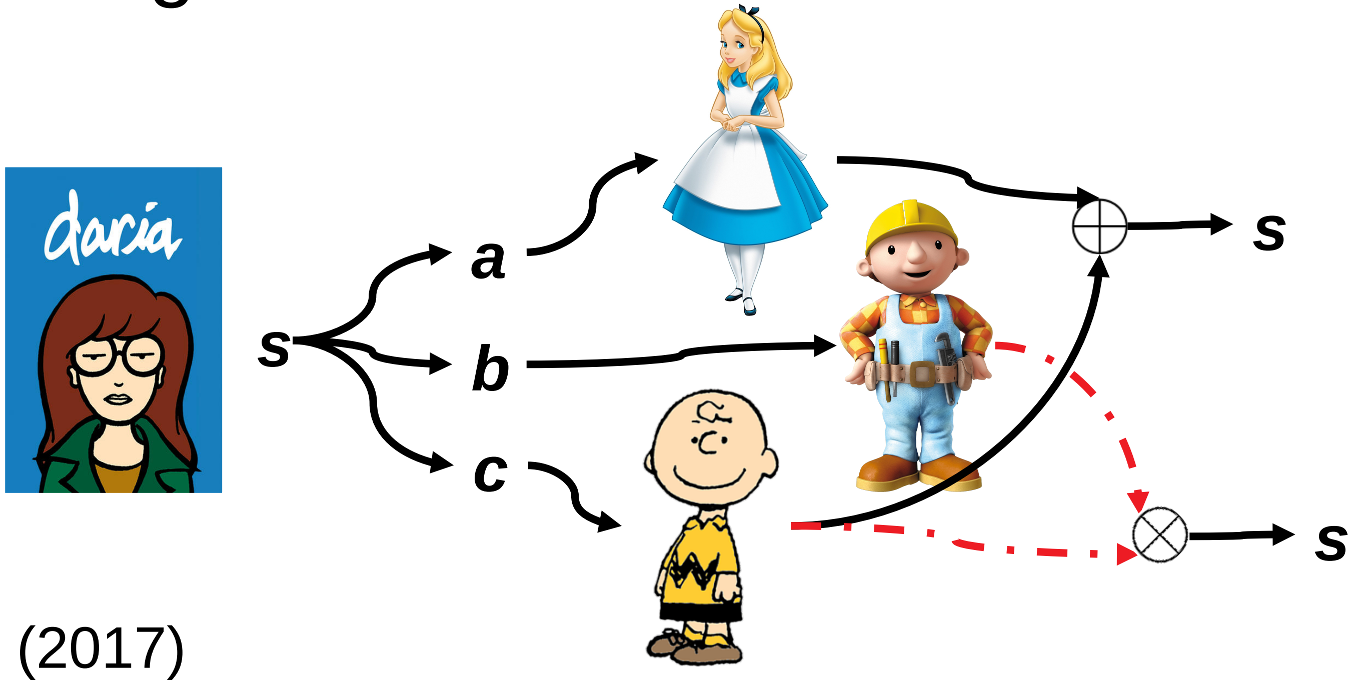

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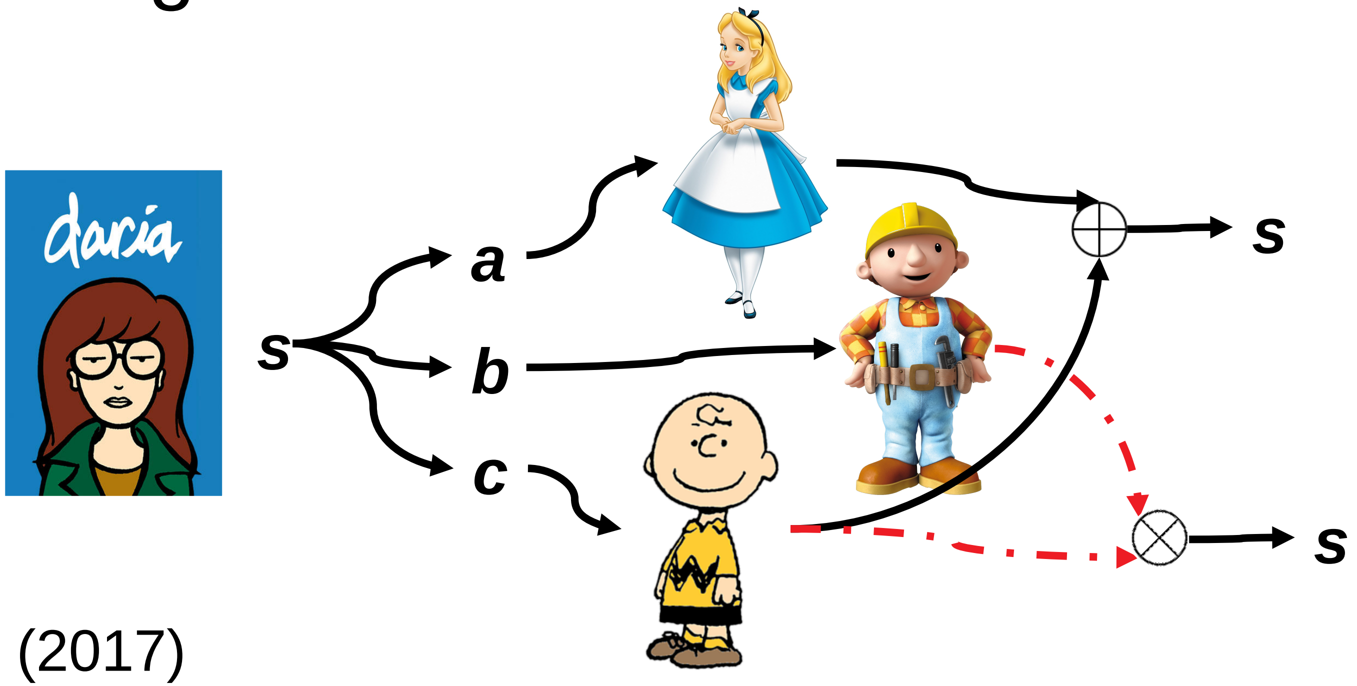

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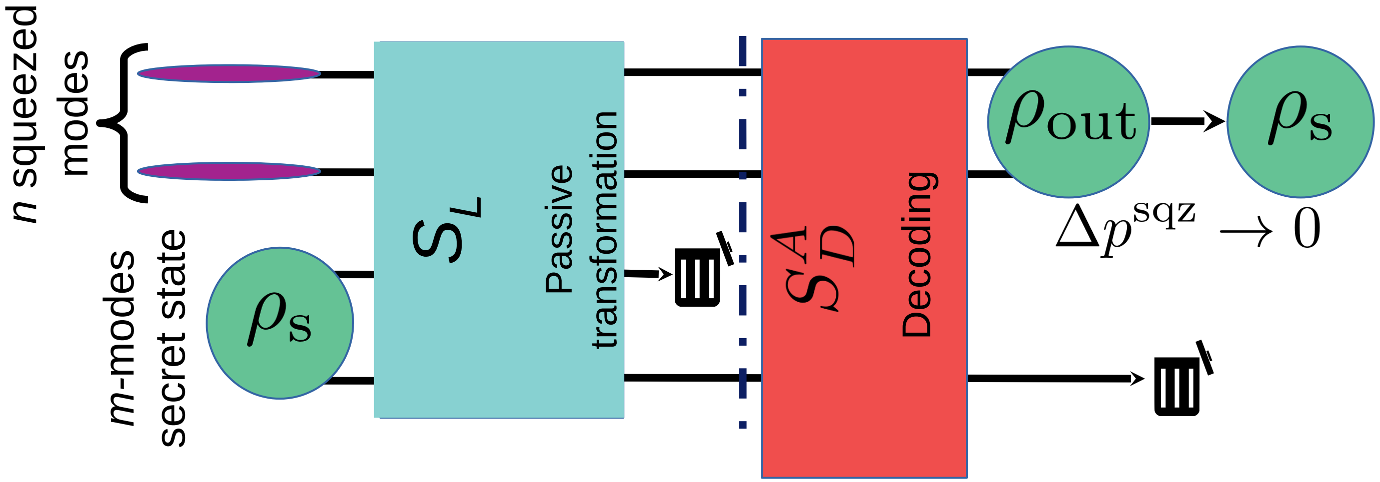
General CV (Gaussian) scheme



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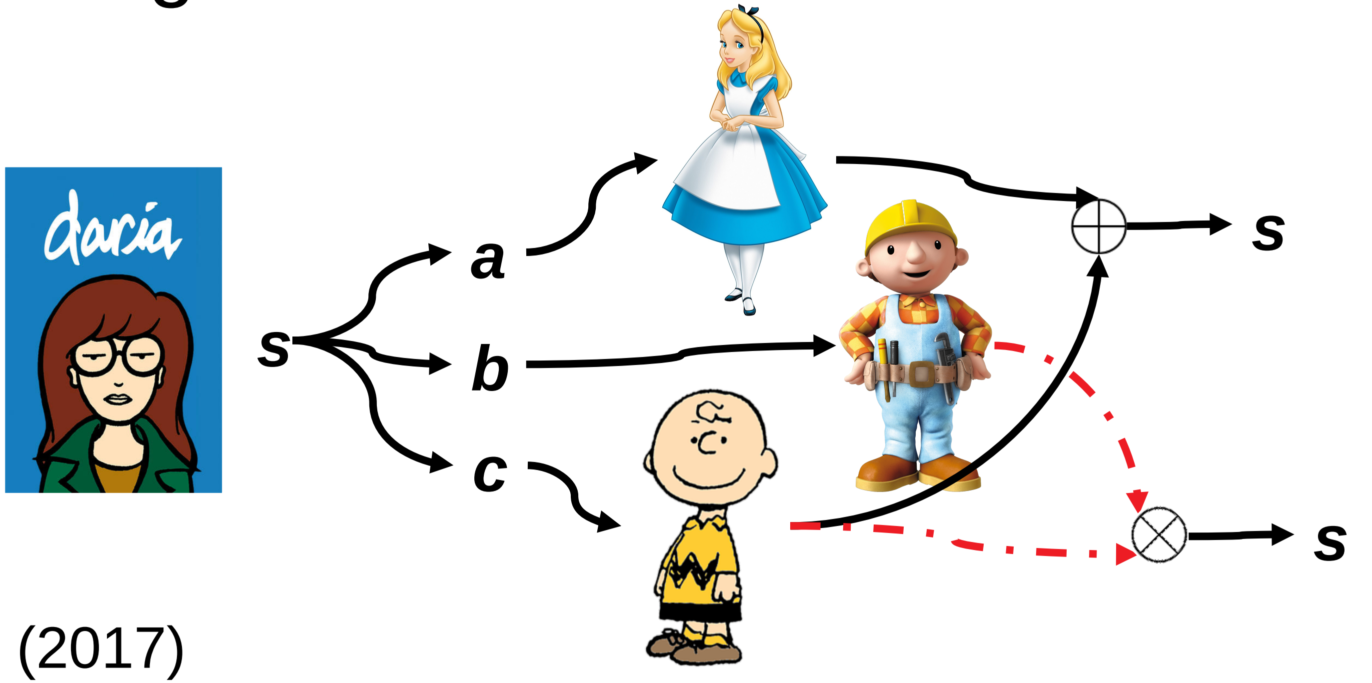

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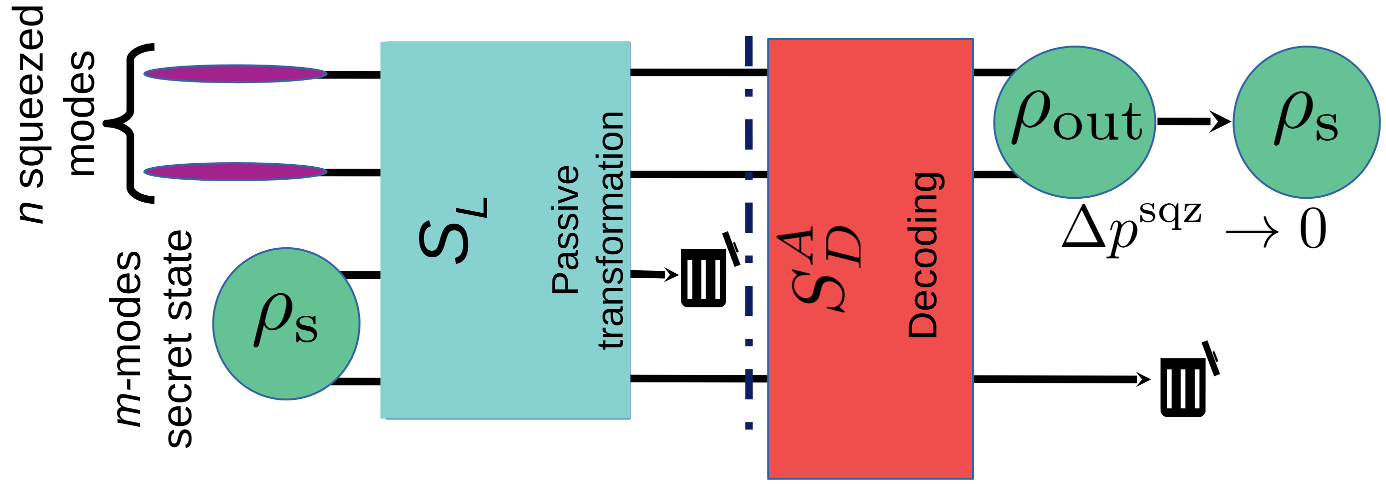


Our contribution:

A CV-QSS scheme can be realized by mixing the secret (quantum) state with squeezed states in **almost any** passive interferometer

Generalizes previous protocols
Experimentally friendly
Analogous to erasure correcting code

Current projects

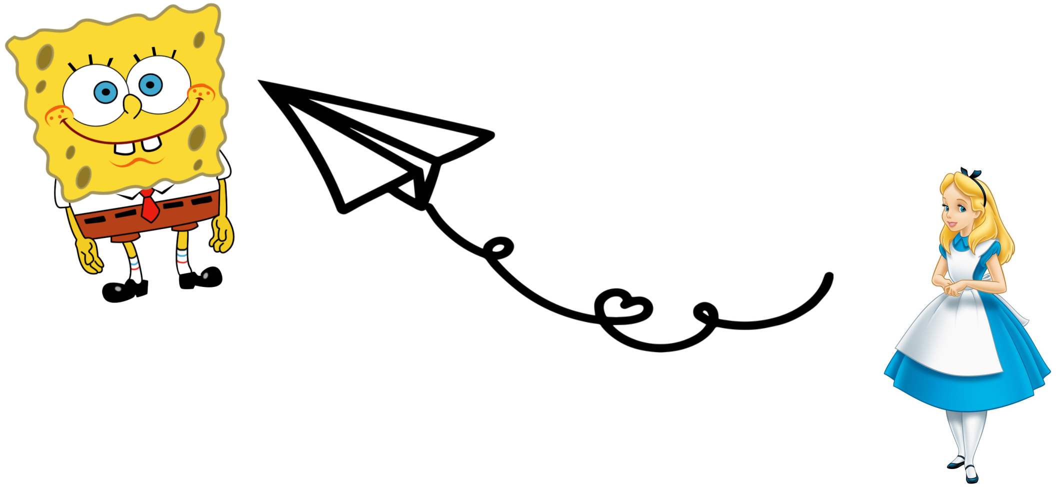
(to appear soon)

With

Jens Eisert

Jonathan Conrad

(Quantum) Error correction and harmonic oscillators



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⇨ Always subject to **noise**

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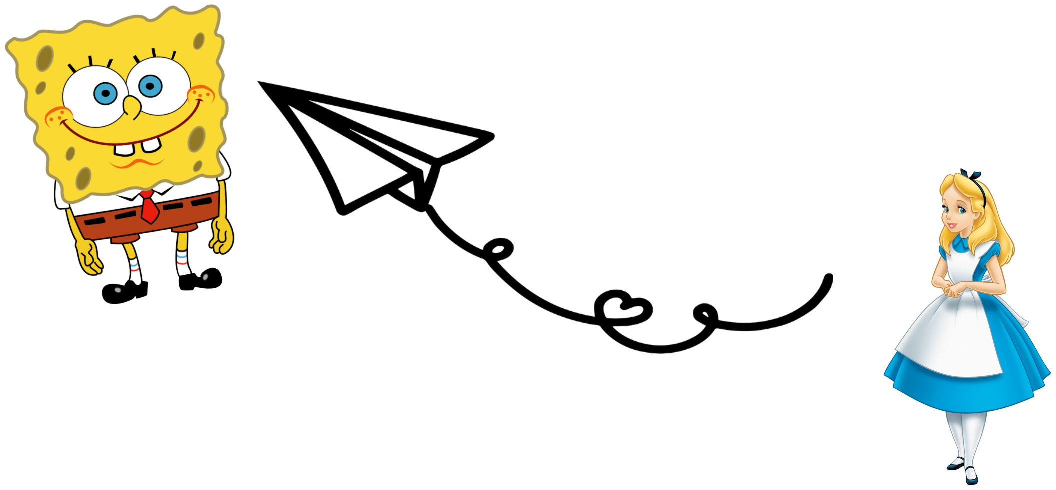

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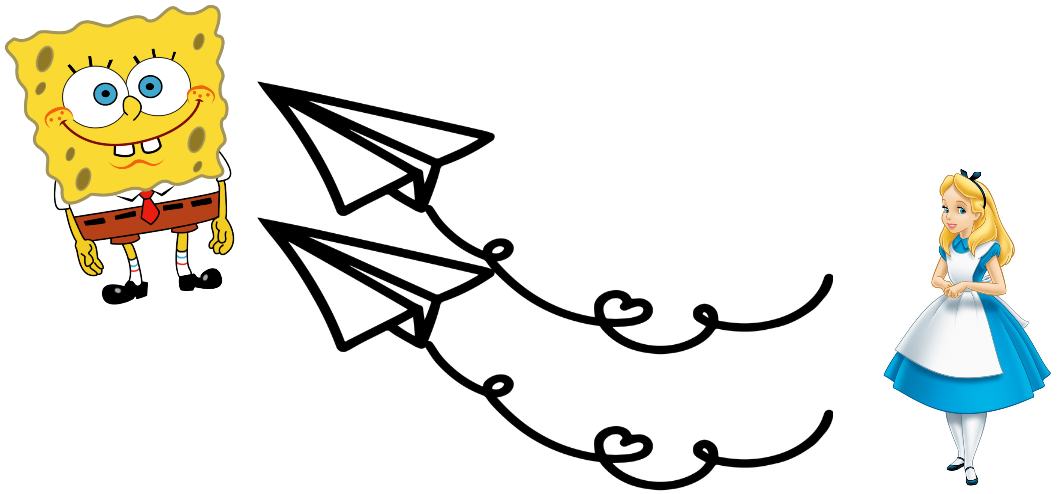

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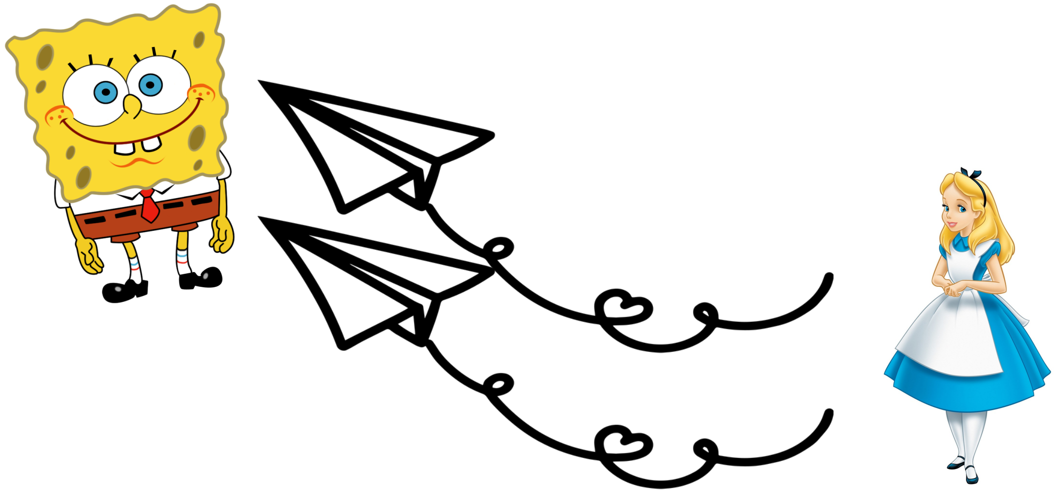

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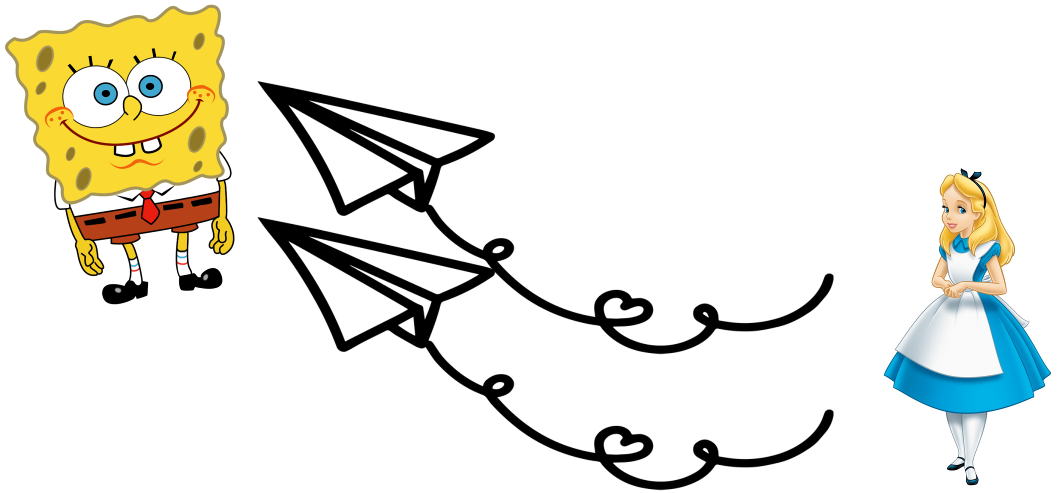

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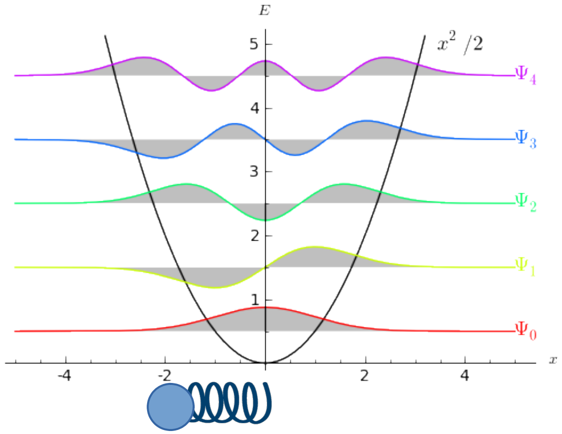
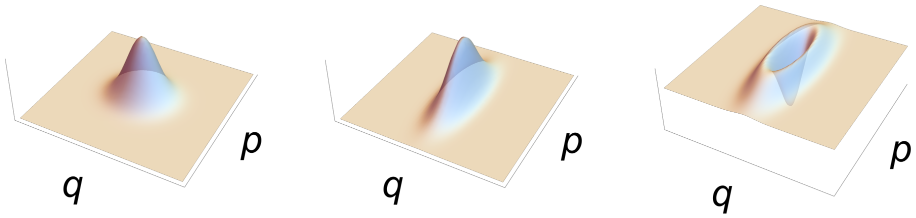


EM field mode, LC circuit, ...

In phase space:
Wigner Function



Quasi-probability distribution

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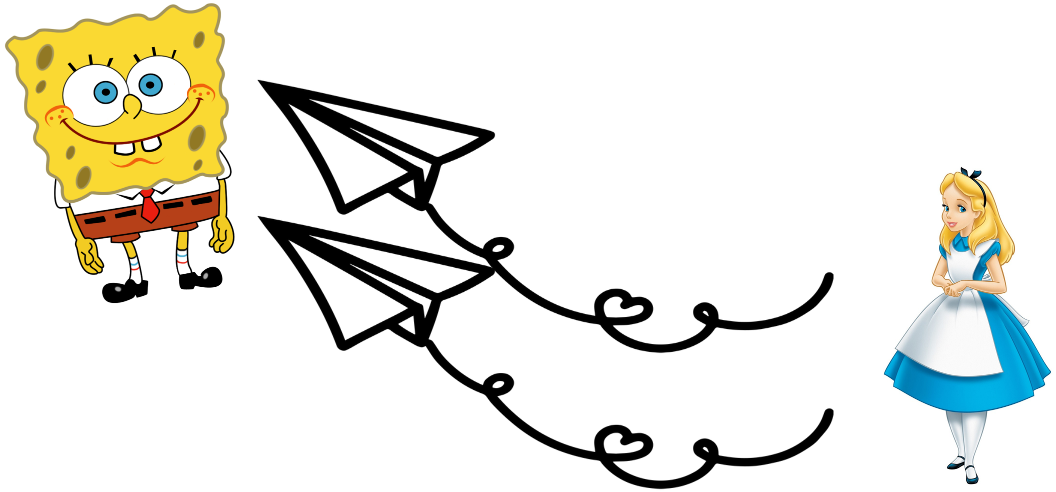

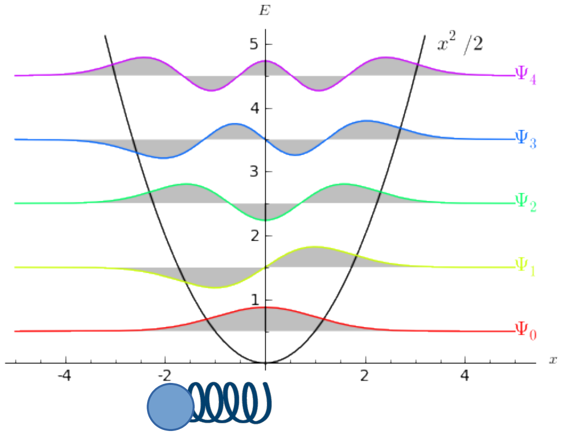

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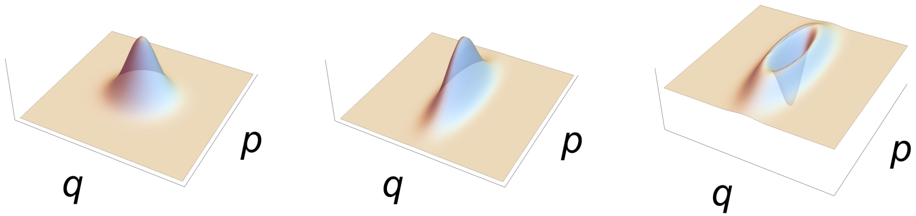

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Infinitely many symbols!
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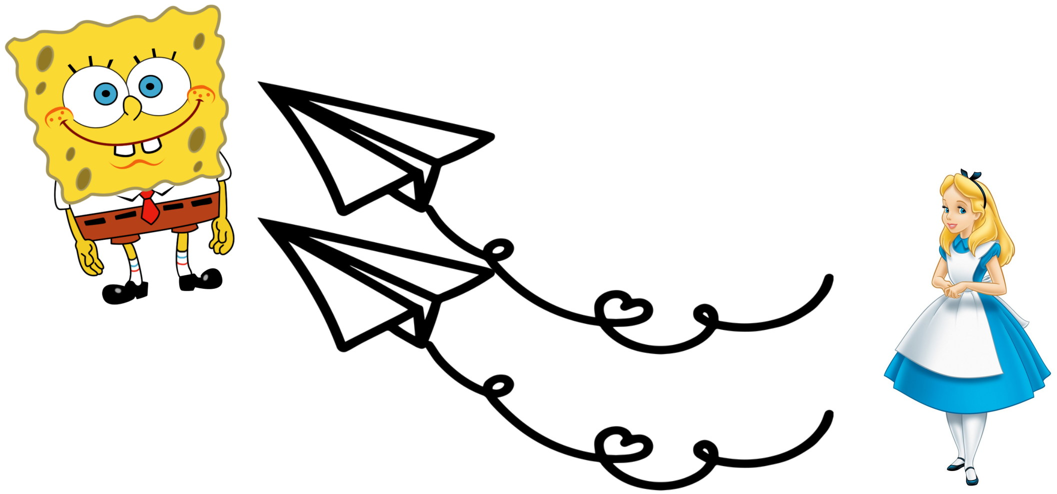

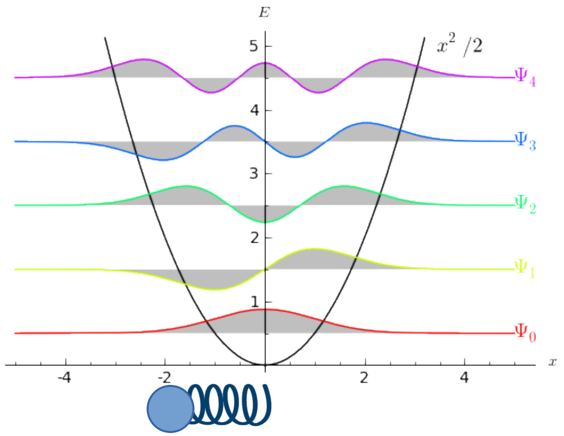

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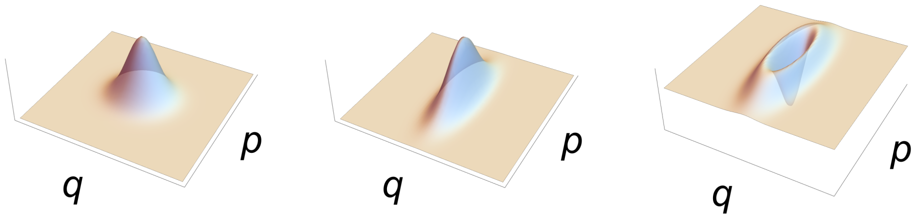

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Symmetries!

Encoding qubits on a lattice

“Logical subspace”: finite photon number, odd/even photon number, or...**translation symmetries**!

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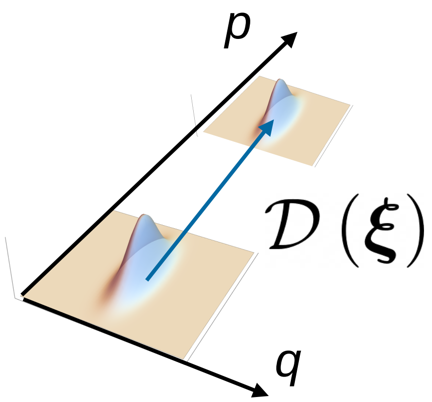
Grid codes: **stabilized** by (commuting) **displacement** operators → underlying **lattice**

Gottesman, Kitaev, Preskill PRA 64 (2001)



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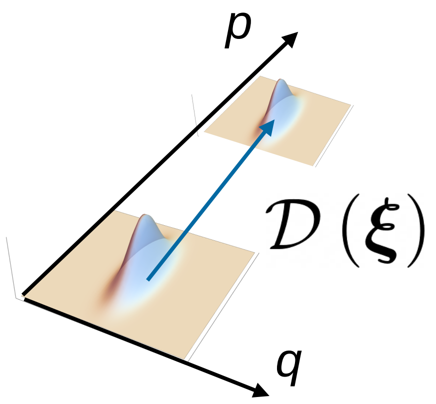

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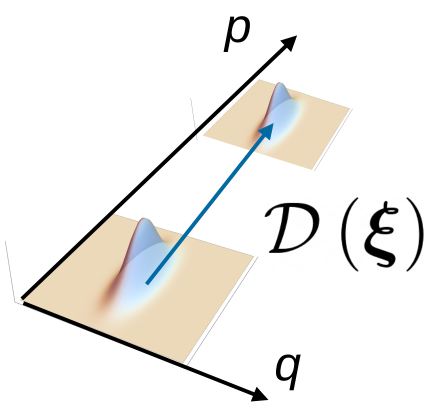

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$\boxed{\mathcal{D}(\boldsymbol{\xi}_j)\,\mathcal{D}(\boldsymbol{\xi}_k) = \mathcal{D}(\boldsymbol{\xi}_j + \boldsymbol{\xi}_k)}$ **(!)**

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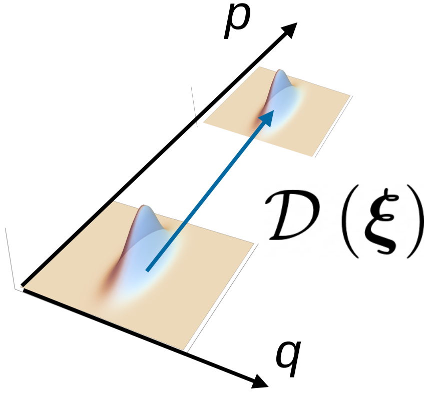

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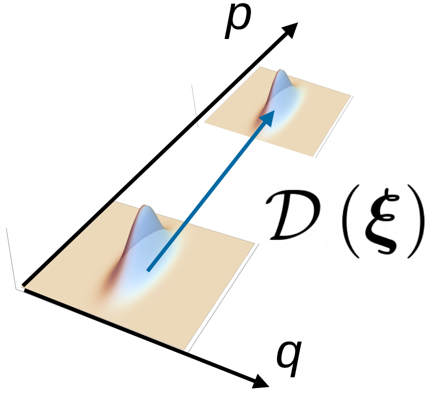

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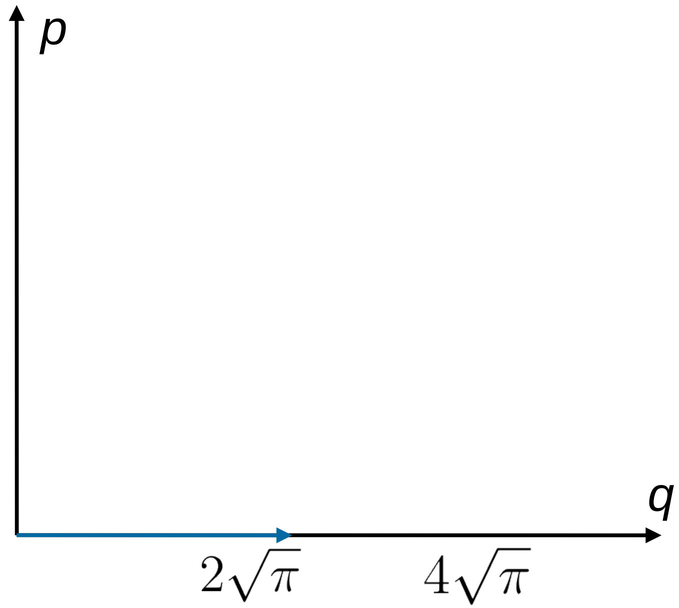

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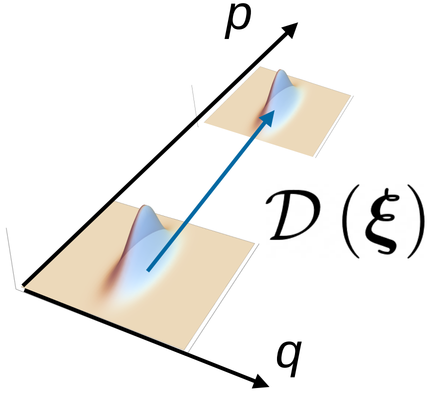

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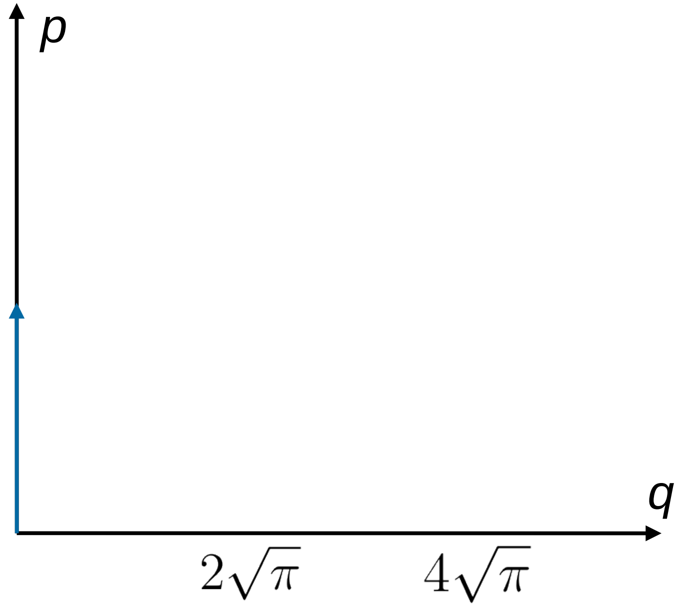

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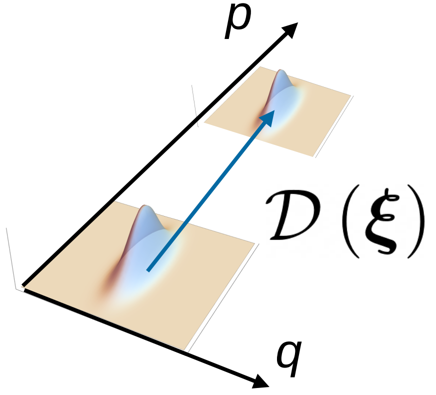

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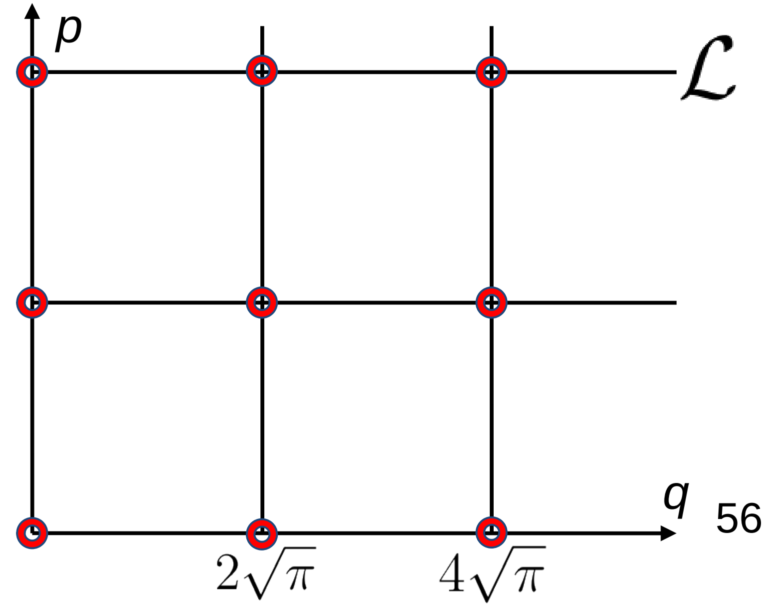

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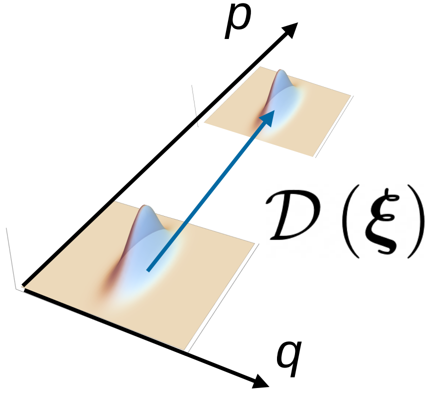

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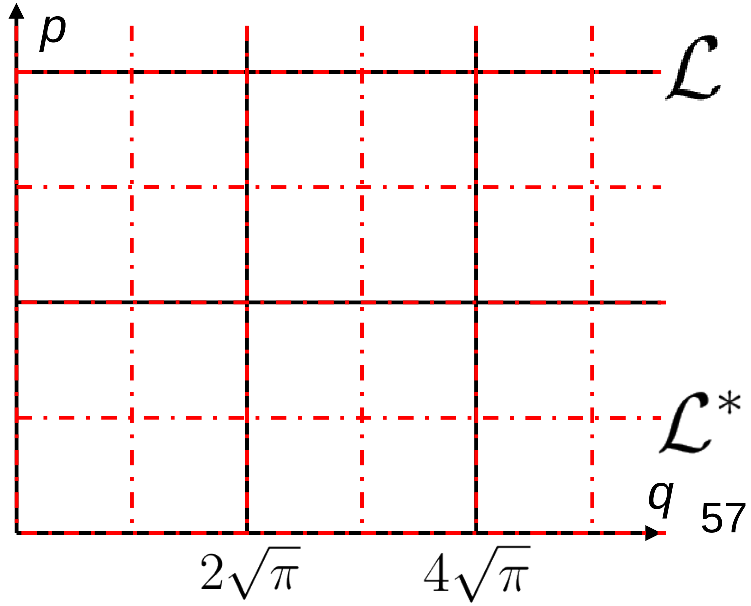

Encoding qubits on a lattice

“Logical subspace”: finite photon number, odd/even photon number, or...**translation symmetries**!

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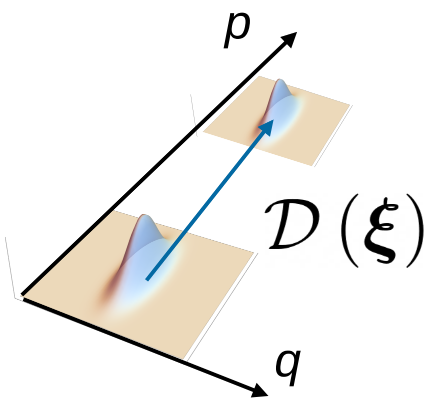
Gottesman, Kitaev, Preskill PRA 64 (2001)



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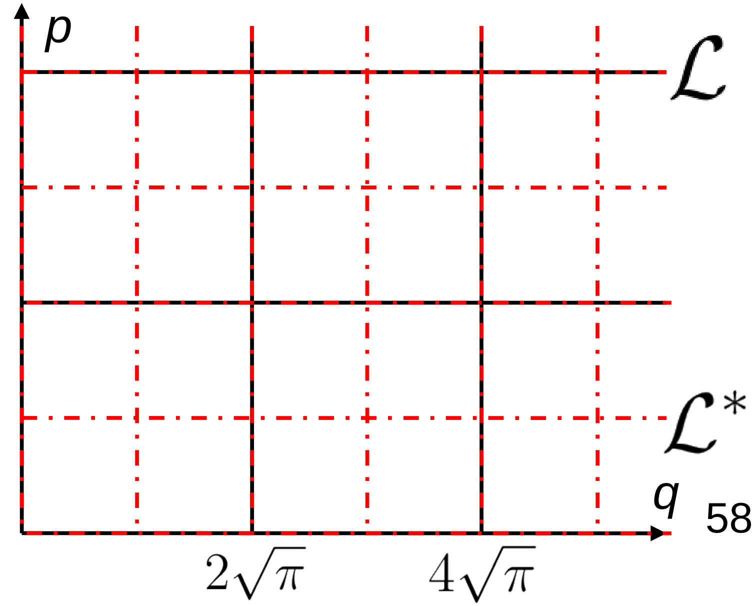

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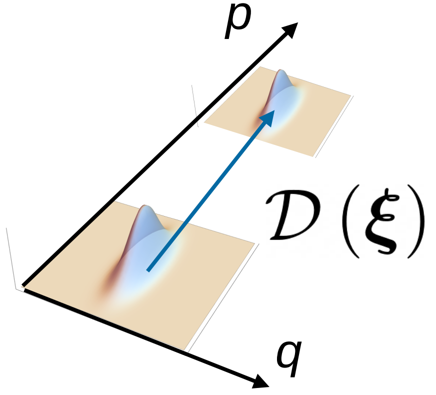

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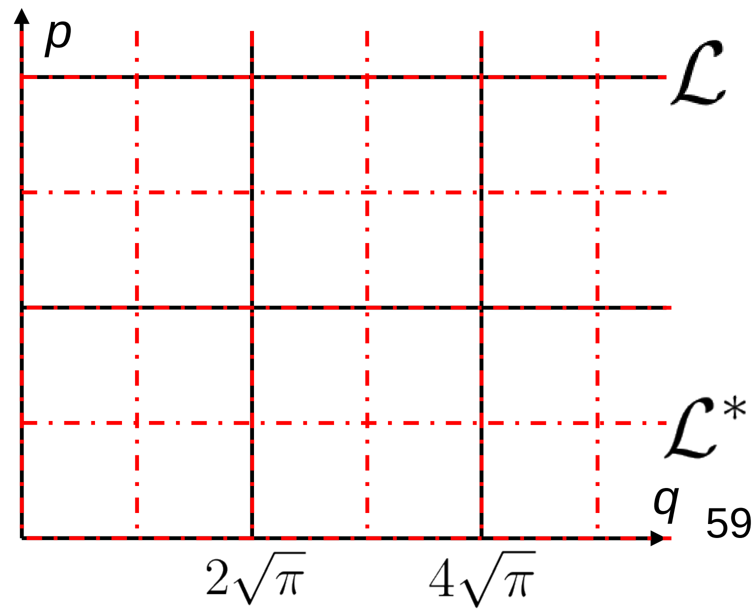

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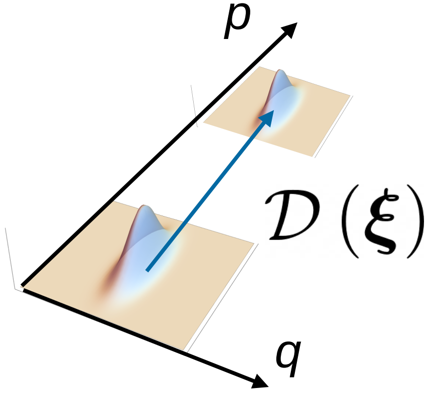

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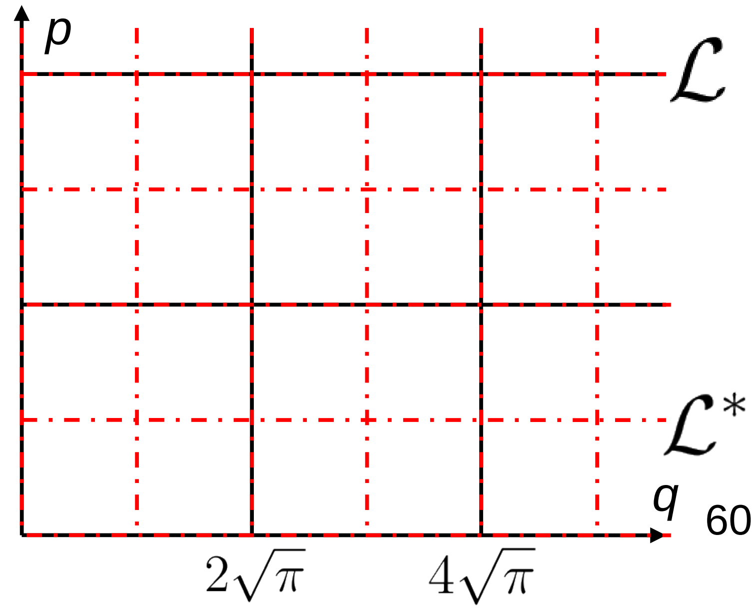

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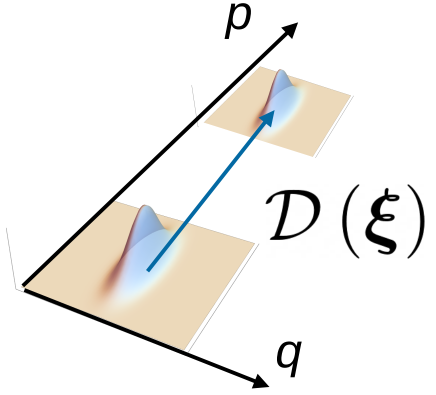

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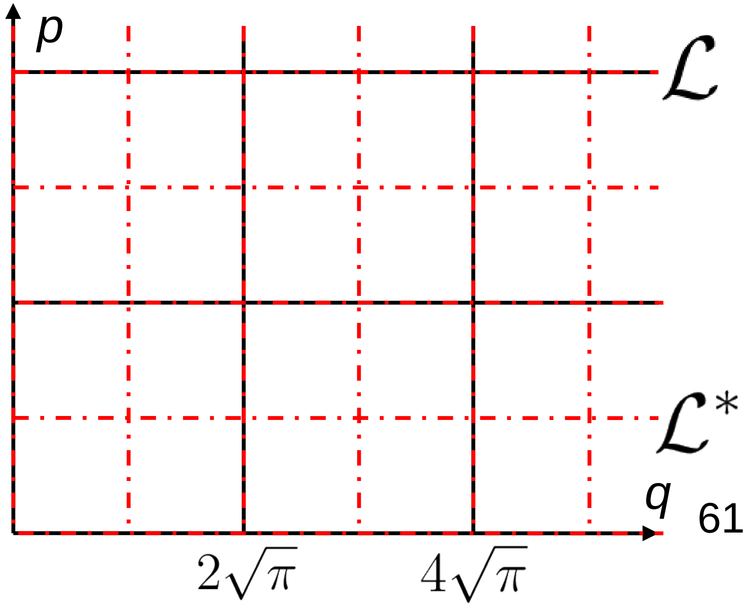

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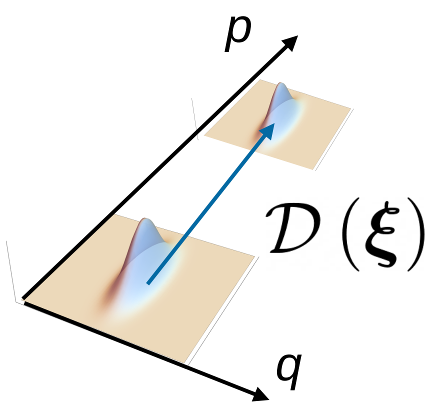

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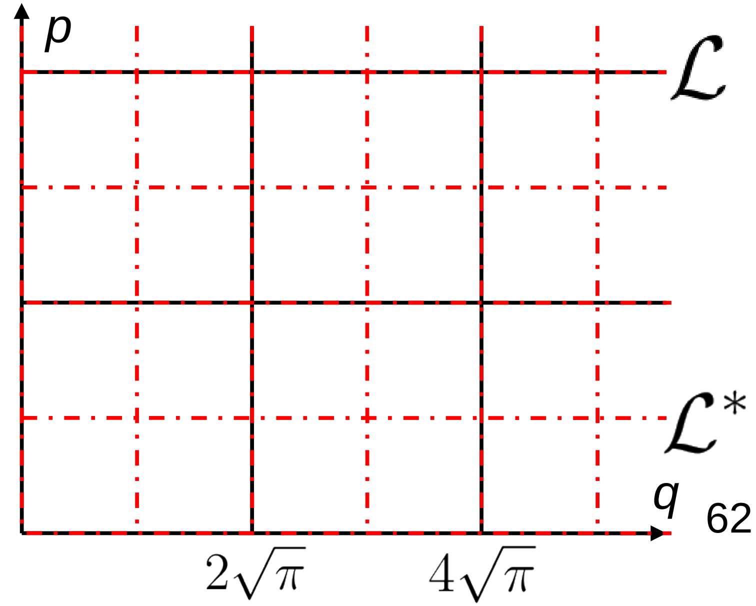

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- Logical states thought hard to realize, now there are experiments!
  Flühmann et al, Nature 566 (2019) *Campagne-Ibarcq et al, Nature 584 (2020)*

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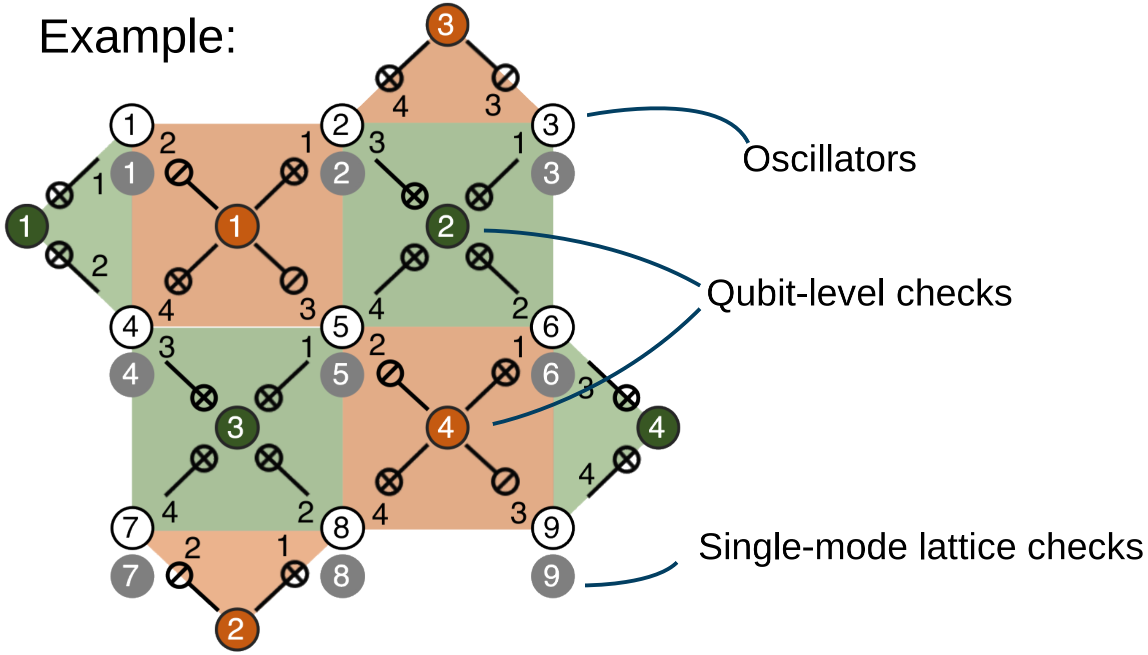
Example:



Oscillators

Qubit-level checks

Single-mode lattice checks

Noh&Chamberland PRA 101 (2020)

$2n = 18$ mode-wise stab.
+
$n-1 = 8$ qubit stab.

Tot = $3n - 1$ = ***26*** stab.

Can achieve same
code with ***18*** total
stab.

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