Continuous-Variable Quantum Computing and Error Correction

Bridging Finite and Infinite Dimensions

Francesco Arzani

QAT

QAT

Quantum

Architectures,
Algorithms,
Applications

and their **T**heory

Overview

Bits : $\psi \in \{0, 1\}$

Qubits : finite dimension → discrete variables

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Continuous variables : infinite state spaces

Also known as *qu-modes*, *bosonic systems*

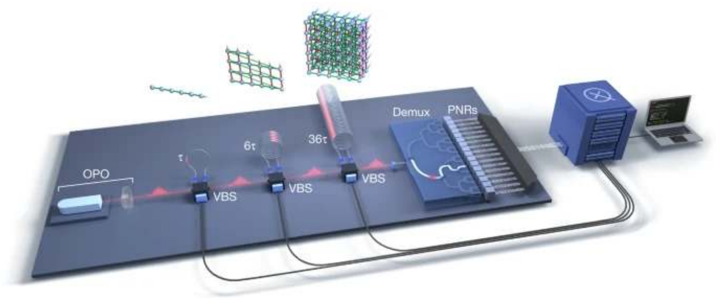
$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + \dots$$

- More general
- Large scale entanglement…

XANADU ALICE & BOB QUANDELA aws

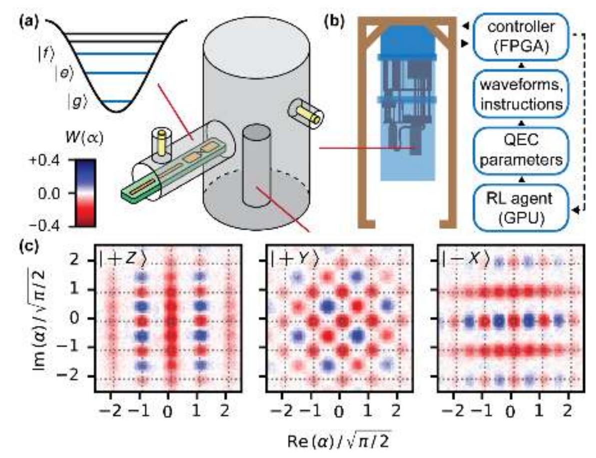
Computational advantage

[MLF+22]



Error correction

[SER+22]



But :
used to be niche ⇒ **Need more theory!**

Finite and infinite systems

Computing

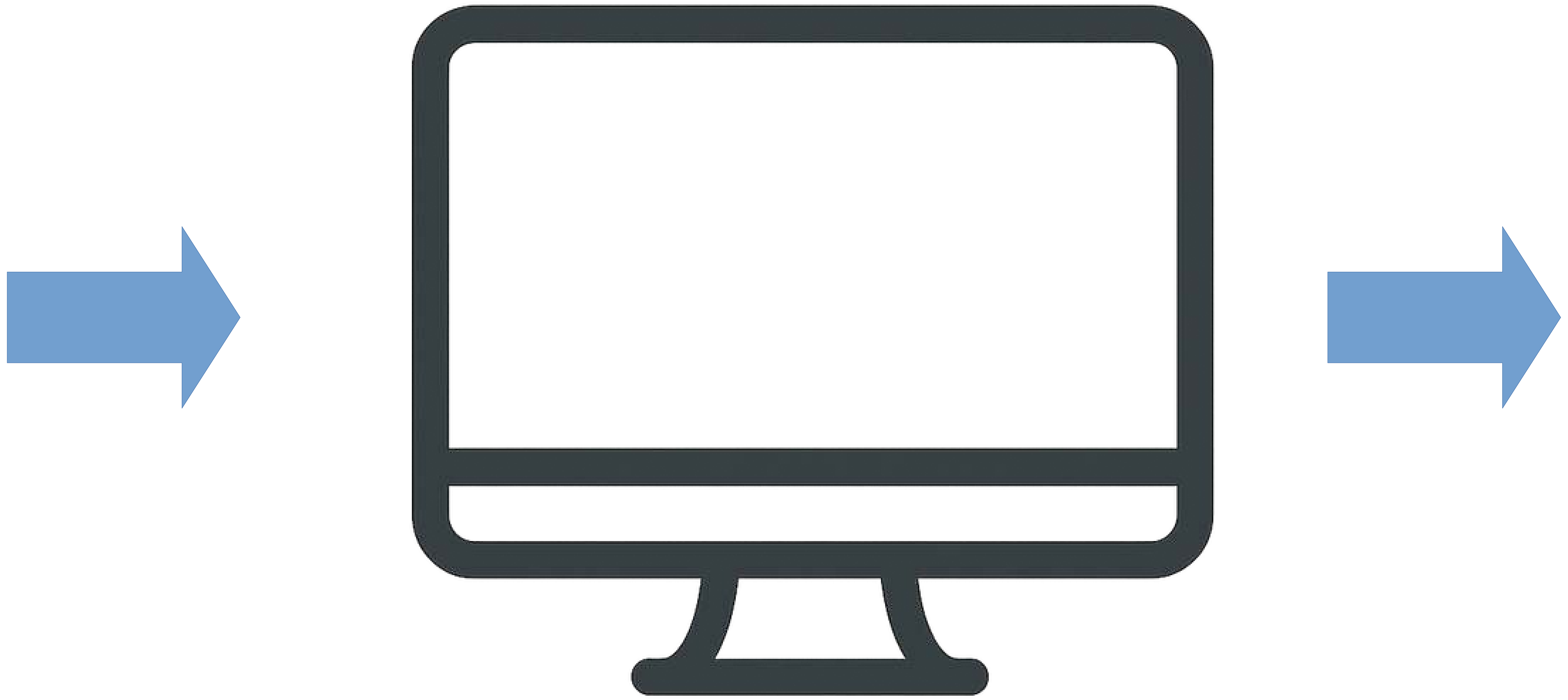
Inputs

$A = 1$

$B = 0$

$C = 1$

$D = 1$

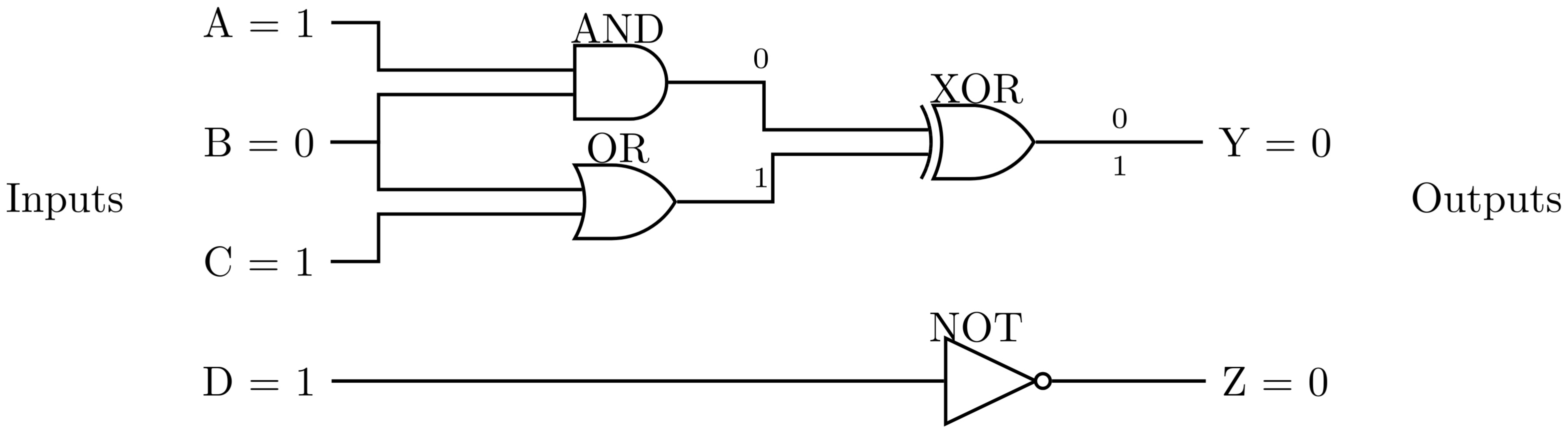


$Y = 0$

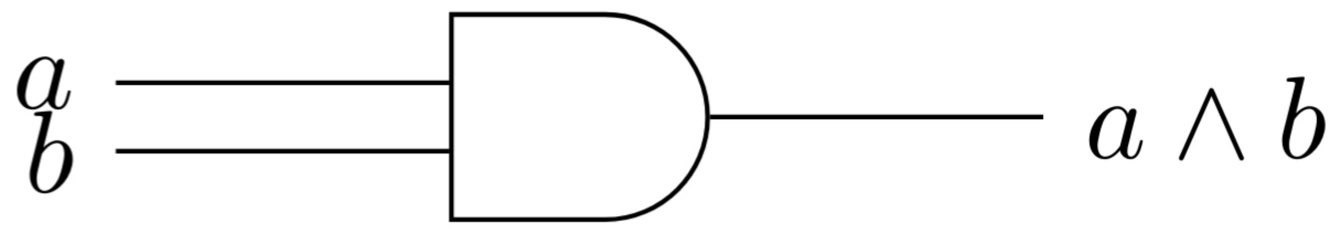
$Z = 0$

Outputs

Computing

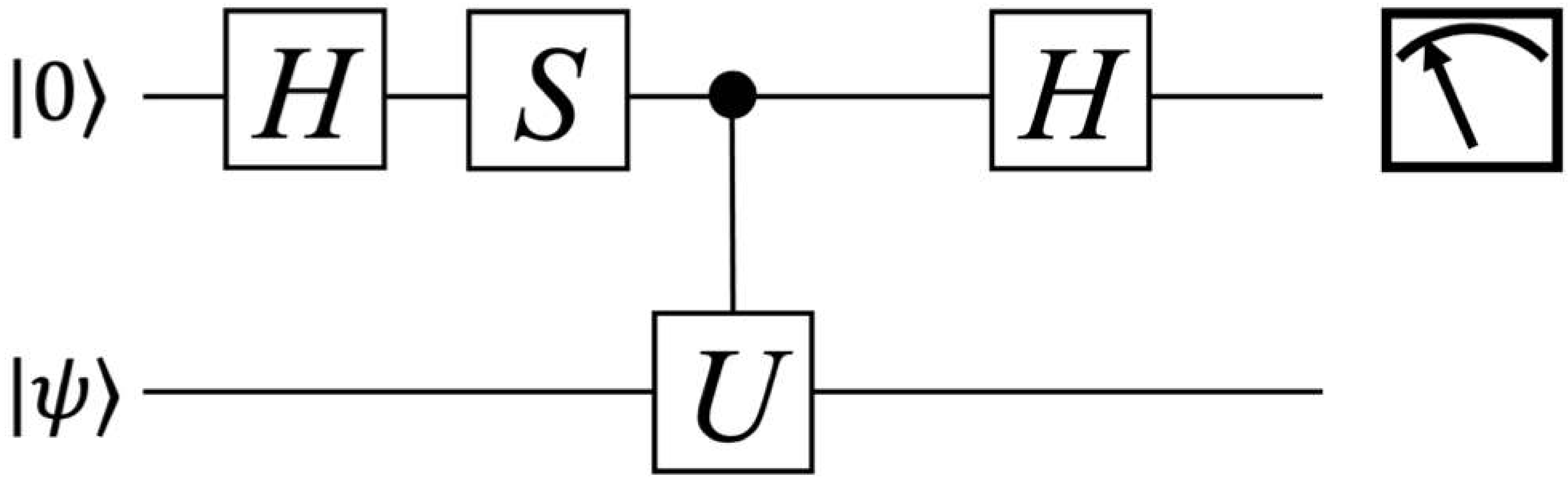


Elementary gates are defined by truth tables



a	**b**	$a \wedge b$
0	0	0
0	1	0
1	0	0
1	1	1

Computing



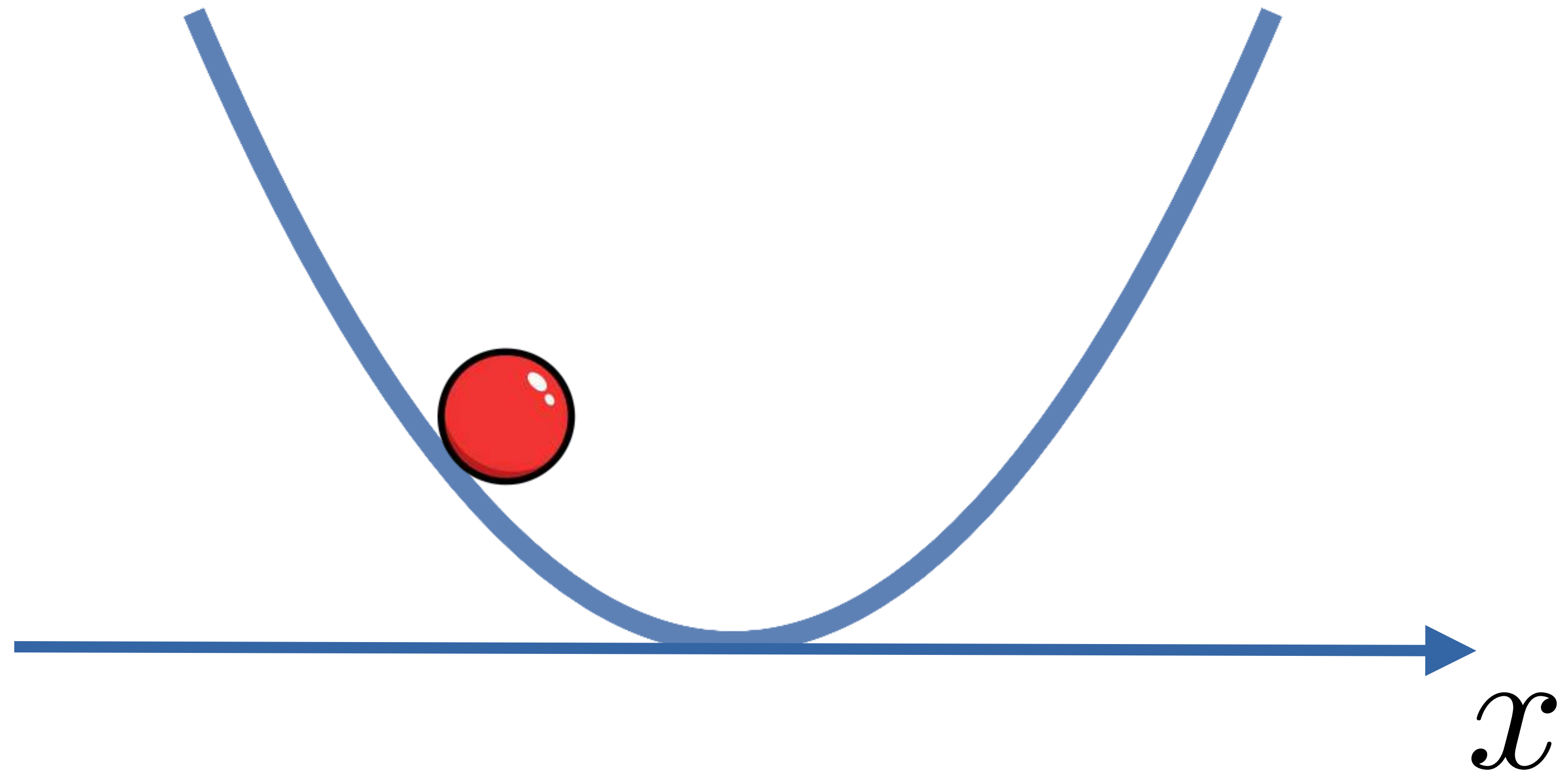
Quantum "truth tables"

$$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

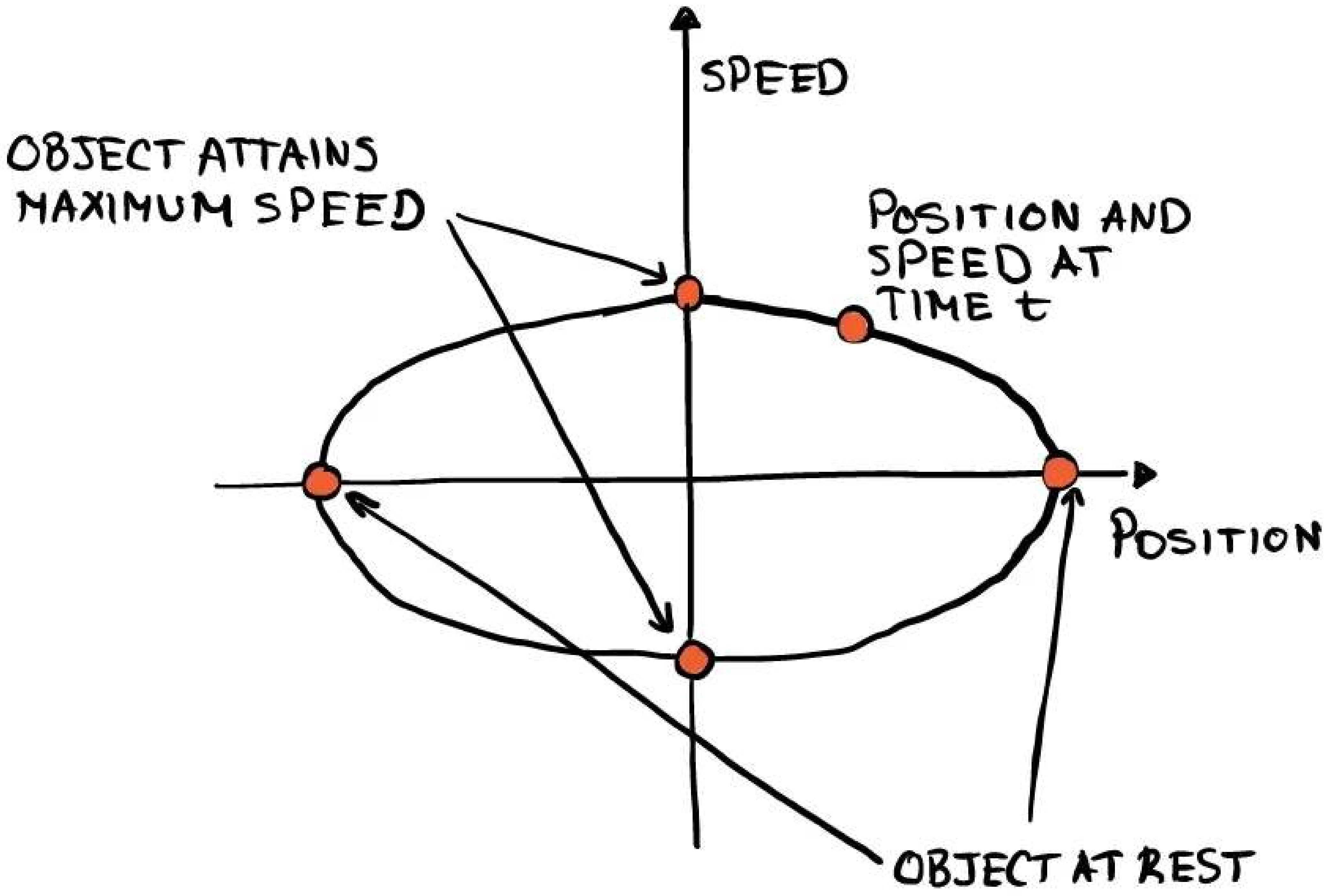
$$H|1\rangle = |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Harmonic oscillators

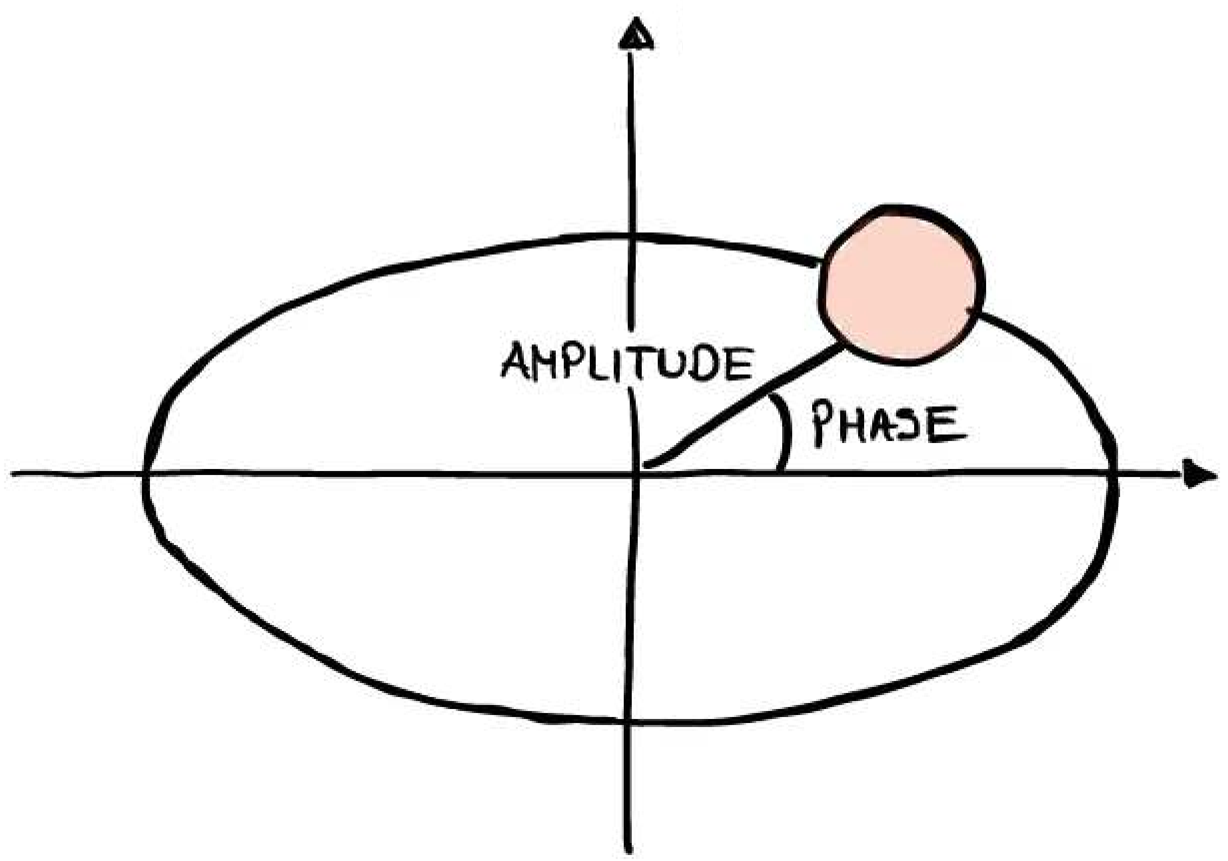
$$\text{Energy} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$



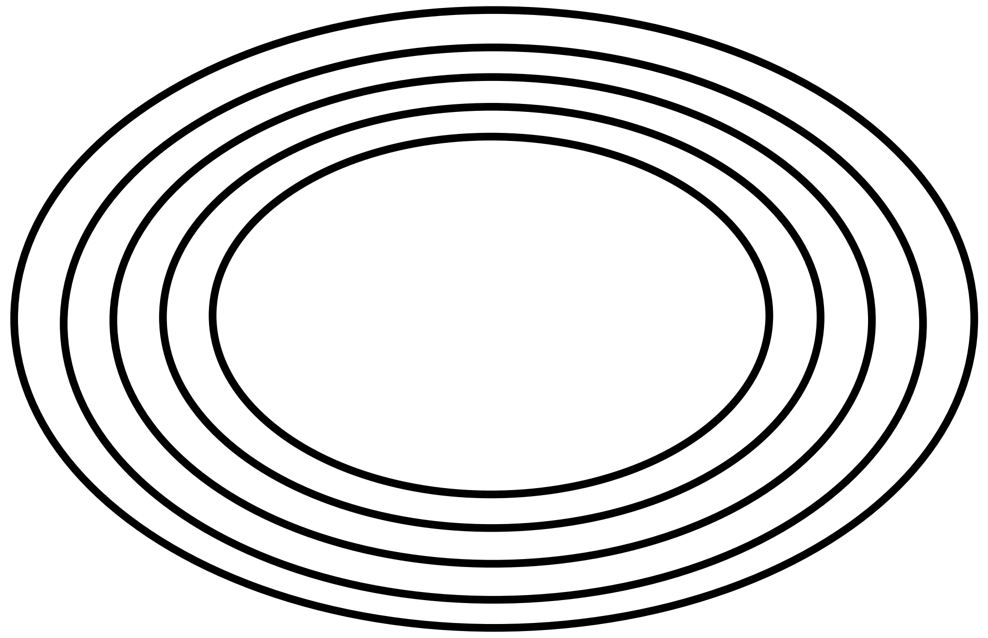
Energy:
Height of the ball
+
How fast it's rolling



Quantum harmonic oscillators



Energy is quantized!



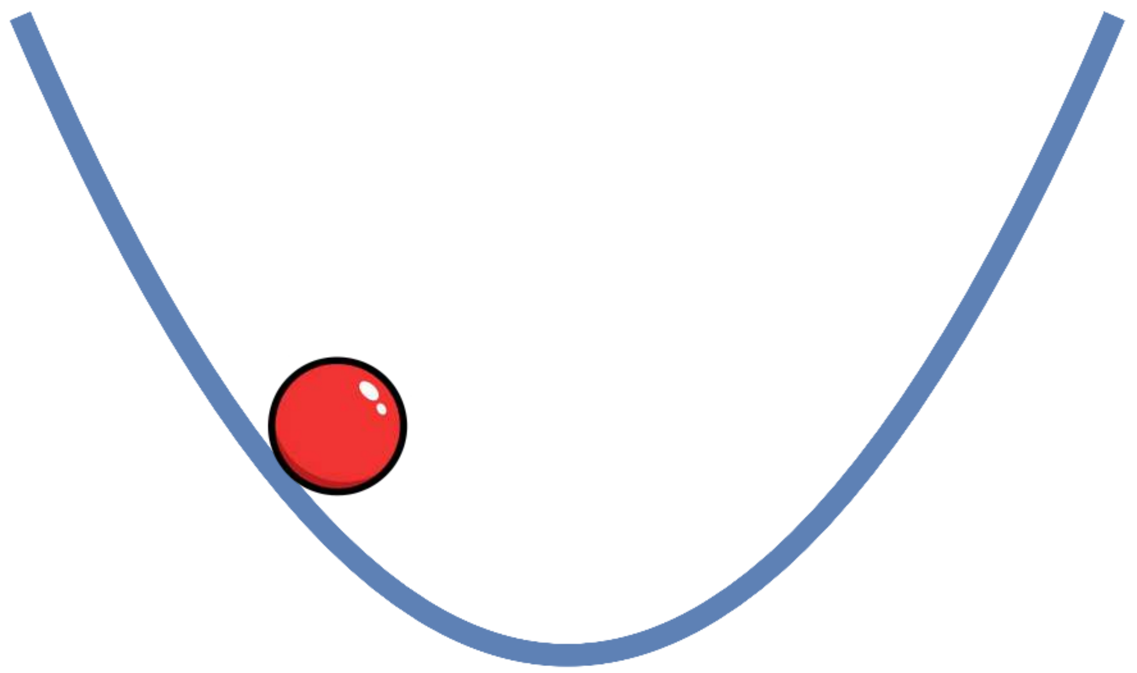
Heisenberg uncertainty principle:

Cannot measure position **and** velocity at the same time

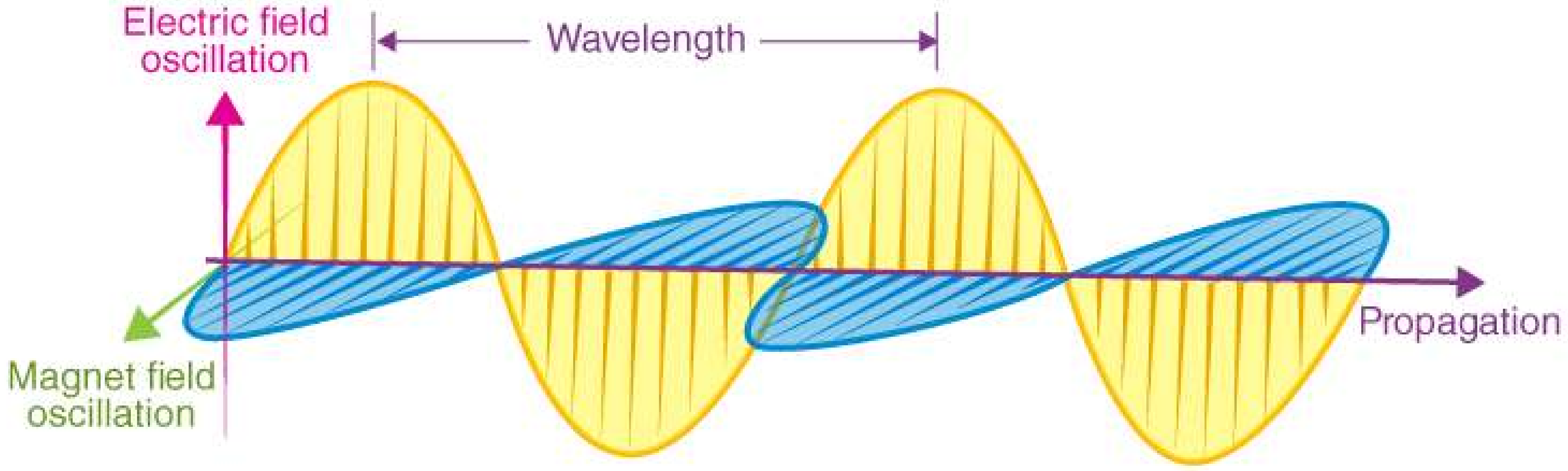
⇨ "circle" instead of a "point"

Quantum harmonic oscillators

$$\text{Energy} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$



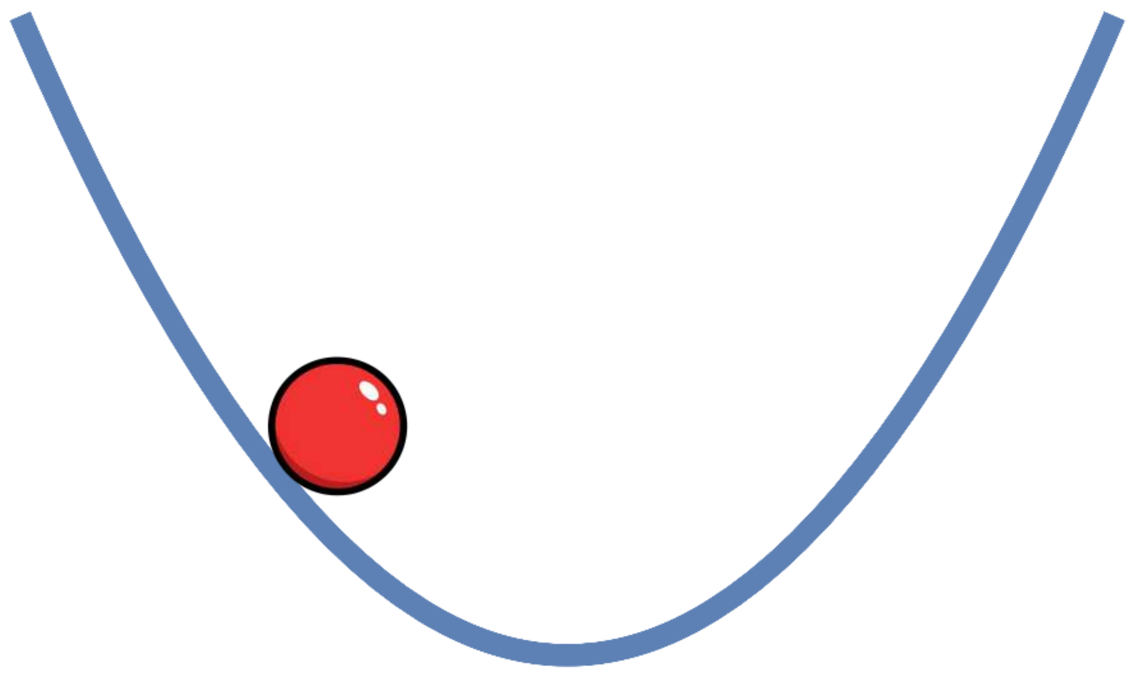
$$\text{Energy} = \frac{\epsilon_0}{2}\hat{\mathbf{E}}^2 + \frac{1}{2\mu_0}\hat{\mathbf{B}}^2$$

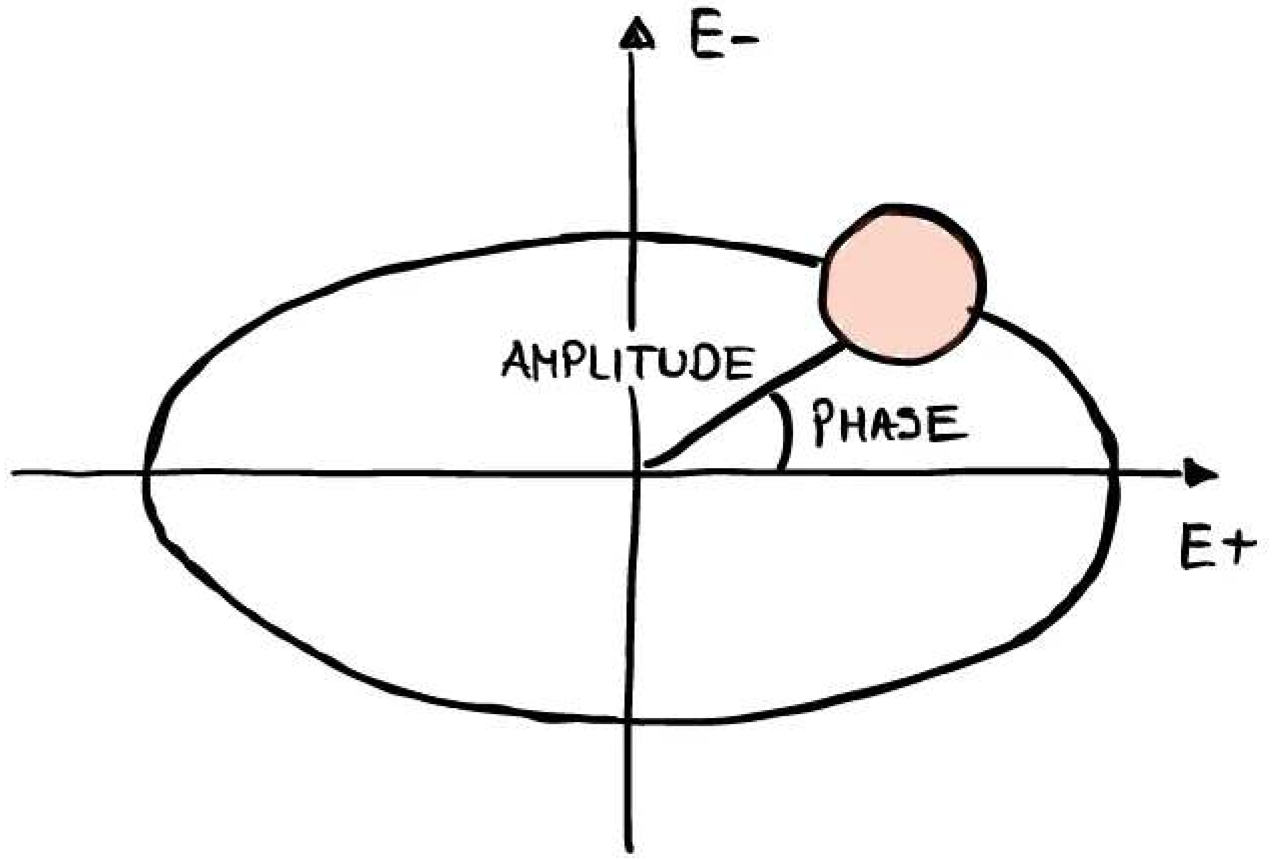


$|1\rangle$

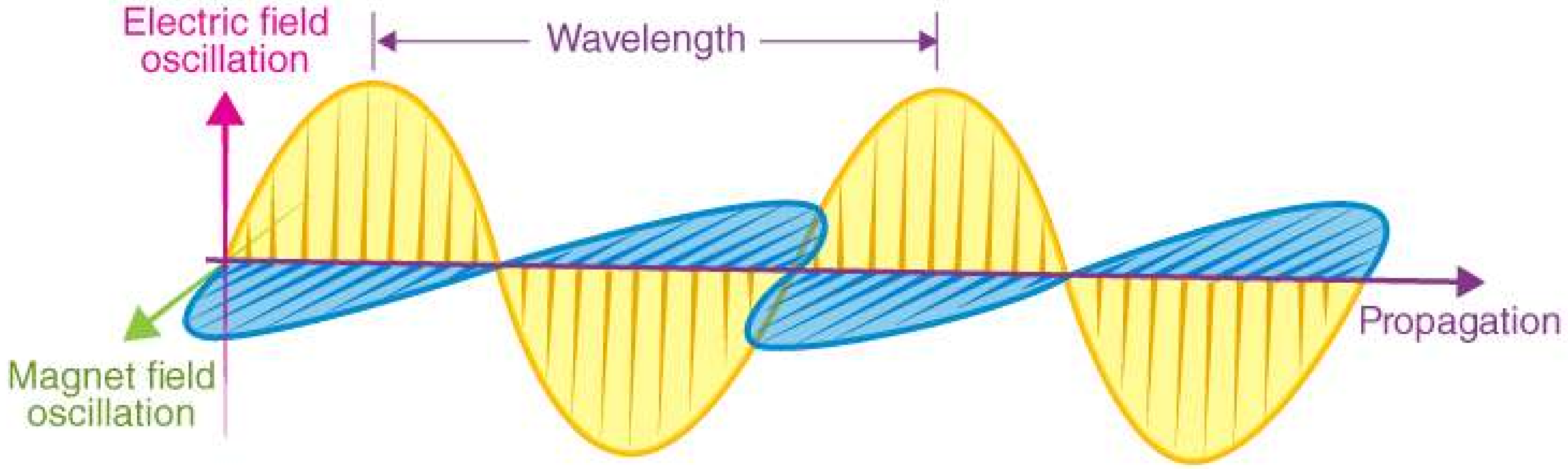
Quantum harmonic oscillators

$$\text{Energy} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

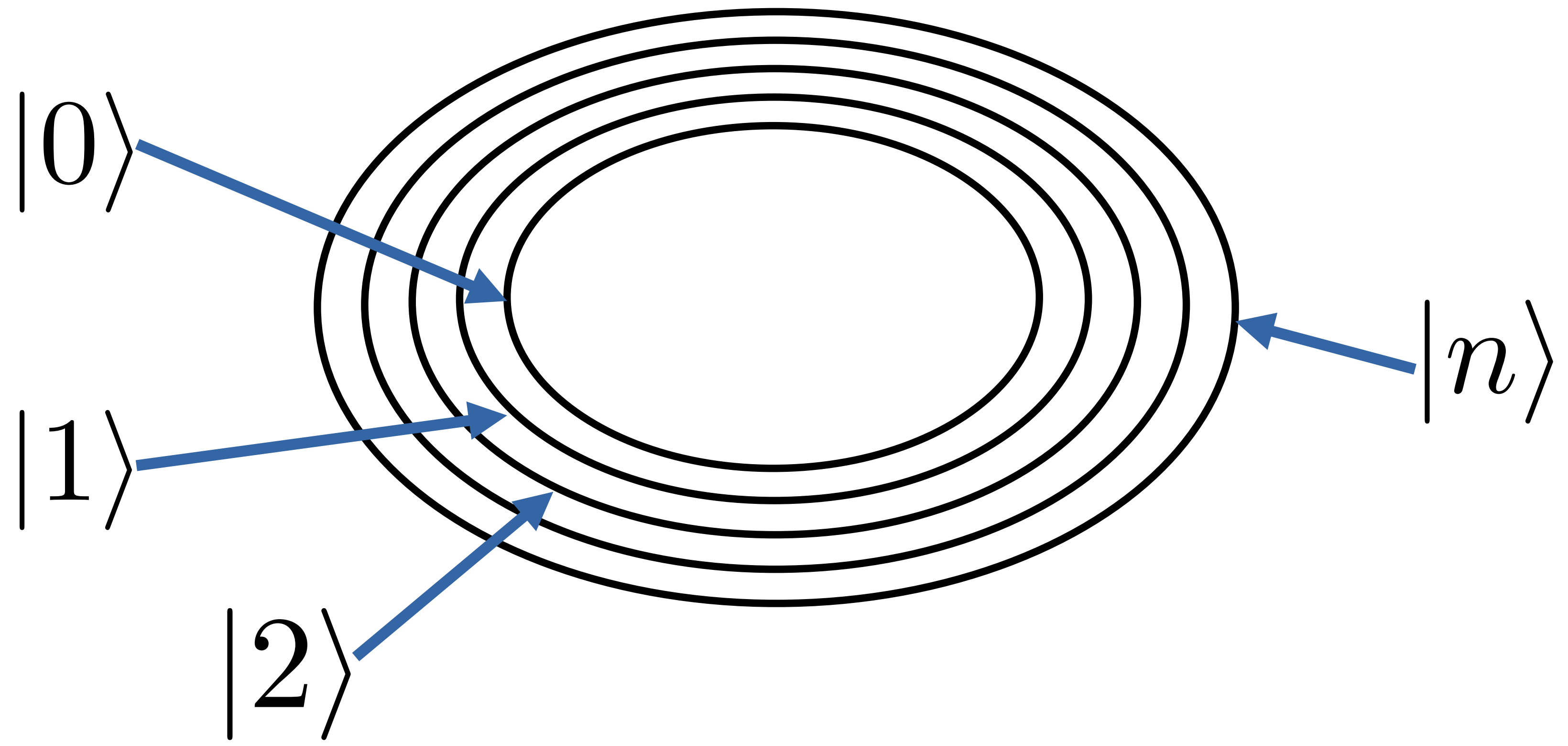




$$\text{Energy} = \frac{\epsilon_0}{2}\hat{\mathbf{E}}^2 + \frac{1}{2\mu_0}\hat{\mathbf{B}}^2$$



Quantized energy = number of photons



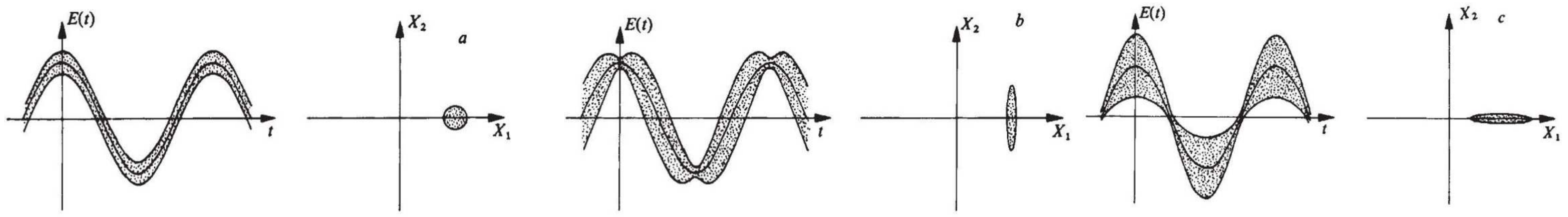
Computing with bosonic systems
(Quantum harmonic oscillators)

Bosonic systems

Bosonic systems appear in many natural and engineered settings
e.g. photonics, circuit QED, trapped ions, cold atoms, ...

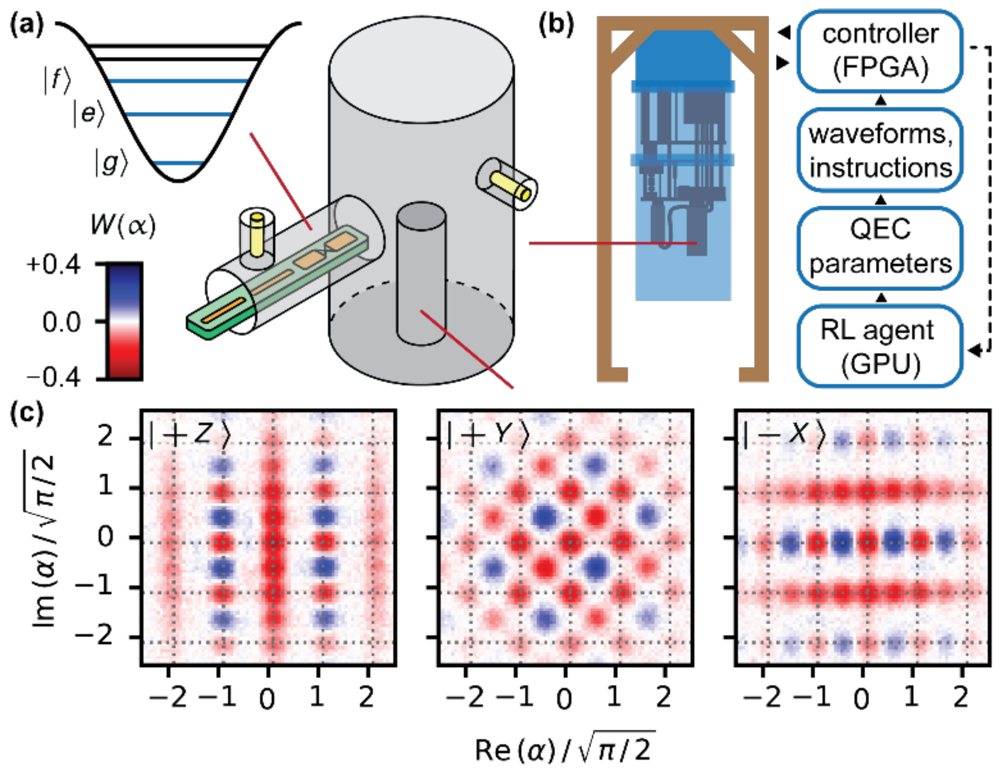
They are described by **infinite dimensional state spaces**.
This gives rise to unique phenomena (and opportunities):

Squeezed states of light



[Walls, Nature 1983]

Bosonic error correction



[Sivak et al, Nature 2023]

Also: gives rise to **mathematical complications**

Often, simplified, **effective descriptions** are used.
Hence the fundamental question :

Can we trust such effective descriptions to capture physical effects?

Yes!

nature communications

Article https://doi.org/10.1038/s41467-025-64872-3

Effective descriptions of bosonic systems can be considered complete

Received: 15 April 2025

Accepted: 30 September 2025

Published online: 06 November 2025

Francesco Arzani[1], Robert I. Booth[2,3] & Ulysse Chabaud[1] ✉

Effective descriptions of bosonic systems

Effective state space

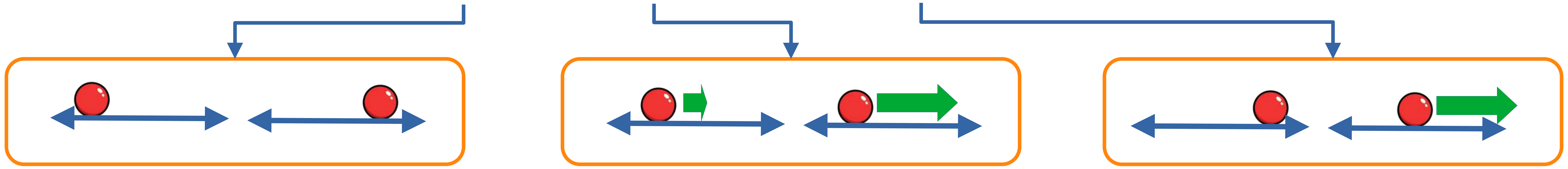
- Truncating the infinite-dimensional state space to a finite *effective dimension*
- Based for instance on energy bounds
- Necessary to simulate bosonic systems on classical and qubit quantum computers

Effective dynamics

- Restricting to specific families of *effective evolutions*
- Used to define continuous-variable quantum computers

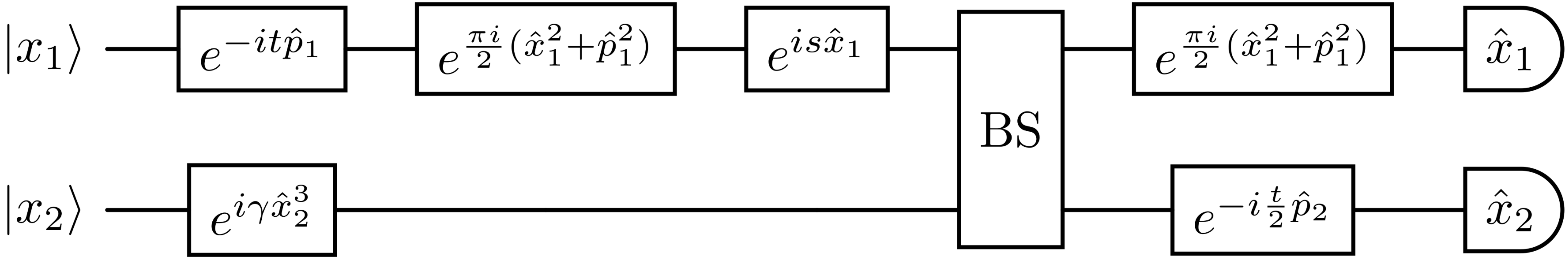
Continuous-variable quantum computing

"Logic" gates: $\left\{e^{-is\hat{p}}, e^{it\hat{x}}, e^{-i\frac{\pi}{2}\left(\hat{x}^2+\hat{p}^2\right)}, e^{i\gamma\hat{x}^3}, e^{i\hat{x}_1\hat{x}_2}\right\}$



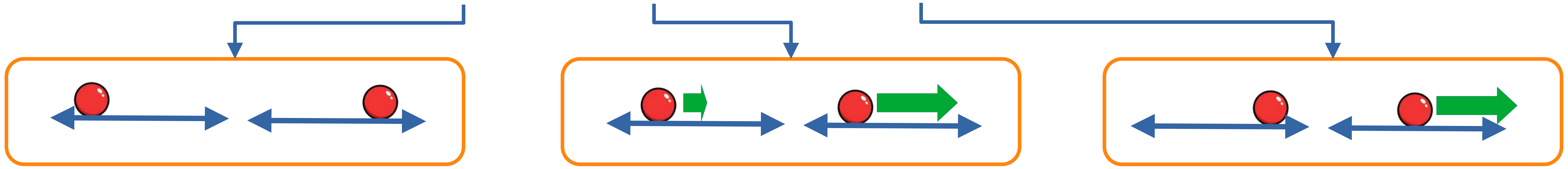
A CV (aka bosonic) QC is a device that can apply gates from this set in any order to a set of oscillators

[Lloyd&Braunstein, PRL 1999]



Continuous-variable quantum computing

"Logic" gates: $\left\{ e^{-is\hat{p}}, e^{it\hat{x}}, e^{-i\frac{\pi}{2}\left(\hat{x}^2+\hat{p}^2\right)}, e^{i\gamma\hat{x}^3}, e^{i\hat{x}_1\hat{x}_2} \right\}$



A CV (aka bosonic) QC is a device that can apply gates from this set in any order to a set of oscillators

[Lloyd&Braunstein, PRL 1999]

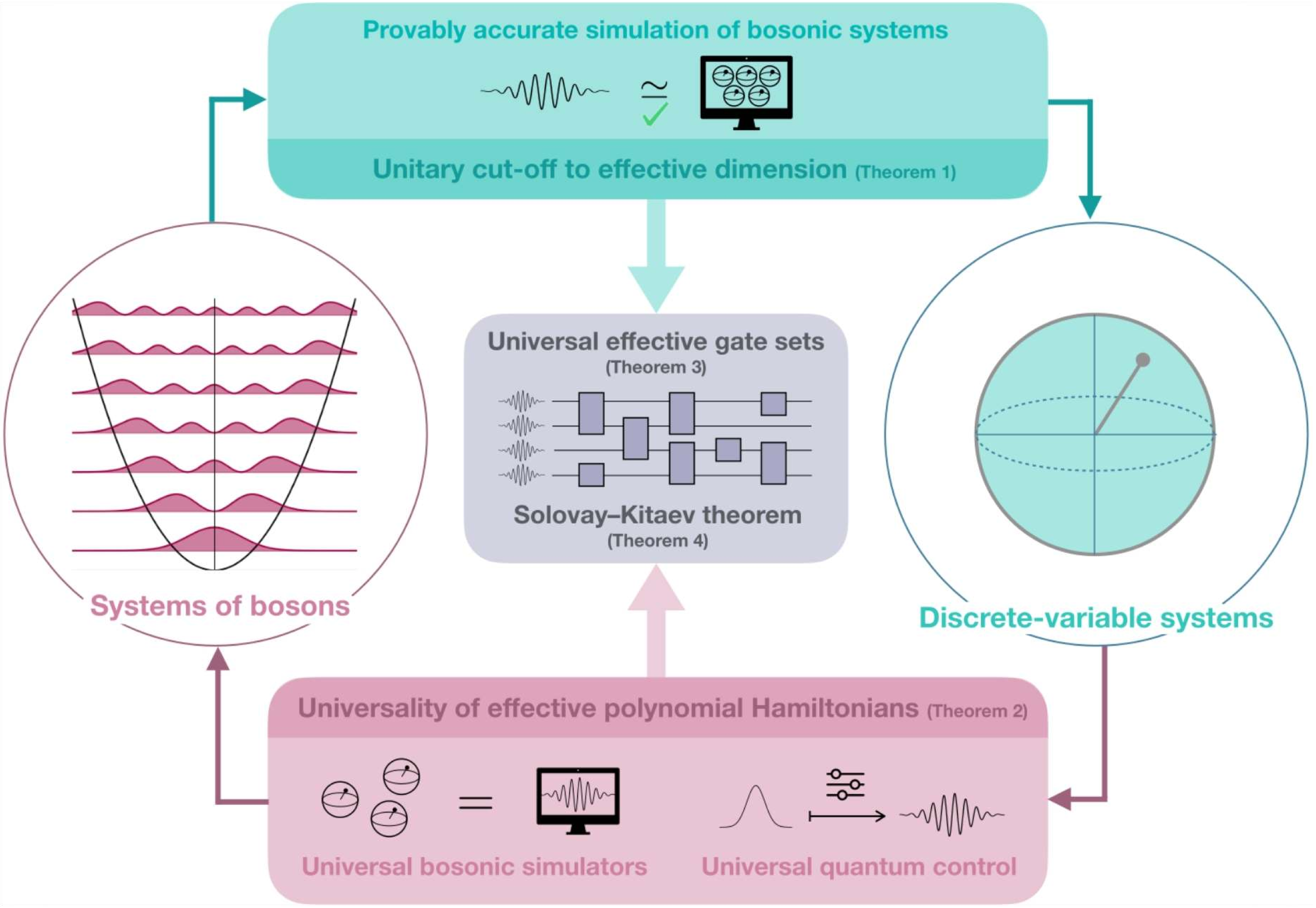
Natural definition but hard to answer questions such as

"How many qubits are needed to simulate the dynamics of a system of bosons to a given accuracy?"
(quantum simulation)

"Are different models of bosonic quantum computers equally powerful?"
(quantum complexity theory)

"Can a universal bosonic computer prepare an arbitrary quantum state?"
(quantum control)

All answered by our work!



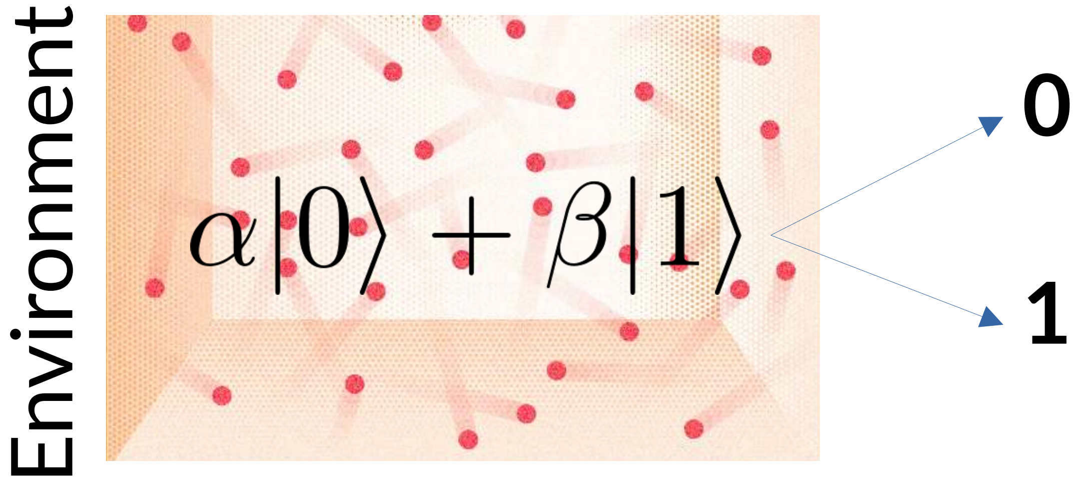
Provably accurate simulation of bosonic systems
Unitary cut-off to effective dimension (Theorem 1)
Universal effective gate sets
(Theorem 3)
Solovay–Kitaev theorem
(Theorem 4)
Systems of bosons
Discrete-variable systems
Universality of effective polynomial Hamiltonians (Theorem 2)
Universal bosonic simulators
Universal quantum control

Error correction

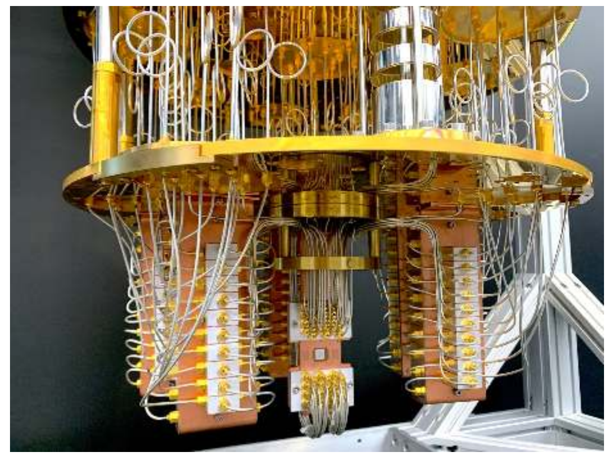
Why?

Quantum information : exploit ***quantum properties***

Problem : these are quickly lost due to **decoherence**



Solution 1 : better machines?



Solution 2 : error correction!

$\rightarrow$ redundancy

Mostly developed for qubits

Peter Shor

$$\alpha|0\rangle + \beta|1\rangle \mapsto \alpha|000\rangle + \beta|111\rangle$$

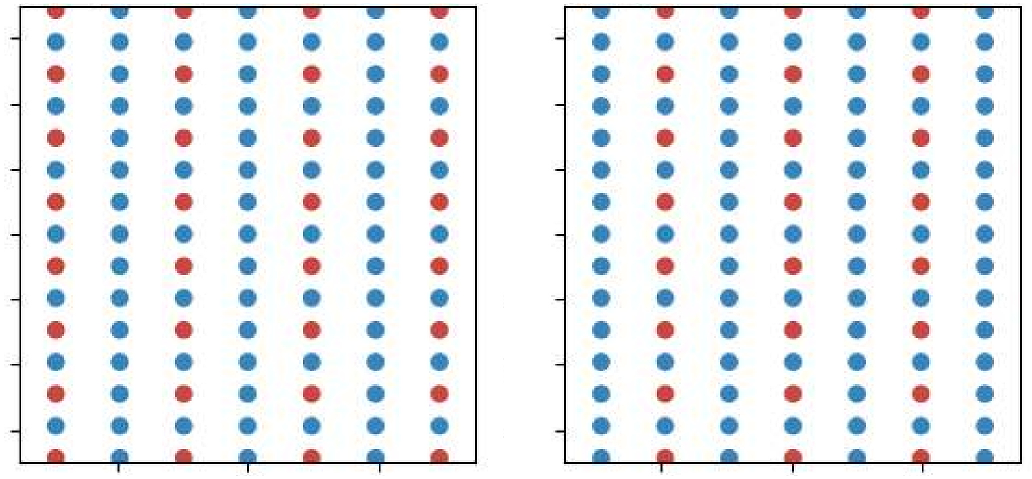
We have *continuous* variables

$$\alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + \alpha_3|3\rangle + \ldots$$

Infinite-dimensional space!
How to exploit?

Symmetries!

$|0_L\rangle$ $|1_L\rangle$



GKP : use lattices

Excursus: symmetries for error correction

source ⇨ encode ⇨ noise ⇨ decode

A (trivial) classical error correcting code: Encode $0 \mapsto 000; \quad 1 \mapsto 111$

Decode: ***majority vote***. Ex: $010 \mapsto 000 \mapsto 0$

Assumption: bits flipped independently at random

A quantum error correcting code? $|0\rangle \mapsto |0\rangle|0\rangle|0\rangle; \quad |1\rangle \mapsto |1\rangle|1\rangle|1\rangle$

By linearity $\alpha|0\rangle + \beta|1\rangle \mapsto \alpha|0\rangle|0\rangle|0\rangle + \beta|1\rangle|1\rangle|1\rangle$

Problem: measuring bit values can alter information !

Measure "flips" instead of bits :

$$Z_l Z_k(\alpha|0\rangle|0\rangle|0\rangle + \beta|1\rangle|1\rangle|1\rangle) = (\alpha|0\rangle|0\rangle|0\rangle + \beta|1\rangle|1\rangle|1\rangle)$$

$$Z_l Z_k(\alpha|0\rangle|1\rangle|0\rangle + \beta|1\rangle|0\rangle|1\rangle) = -(\alpha|0\rangle|1\rangle|0\rangle + \beta|1\rangle|0\rangle|1\rangle) \qquad (l,k) = (1,2),\ (2,3)$$

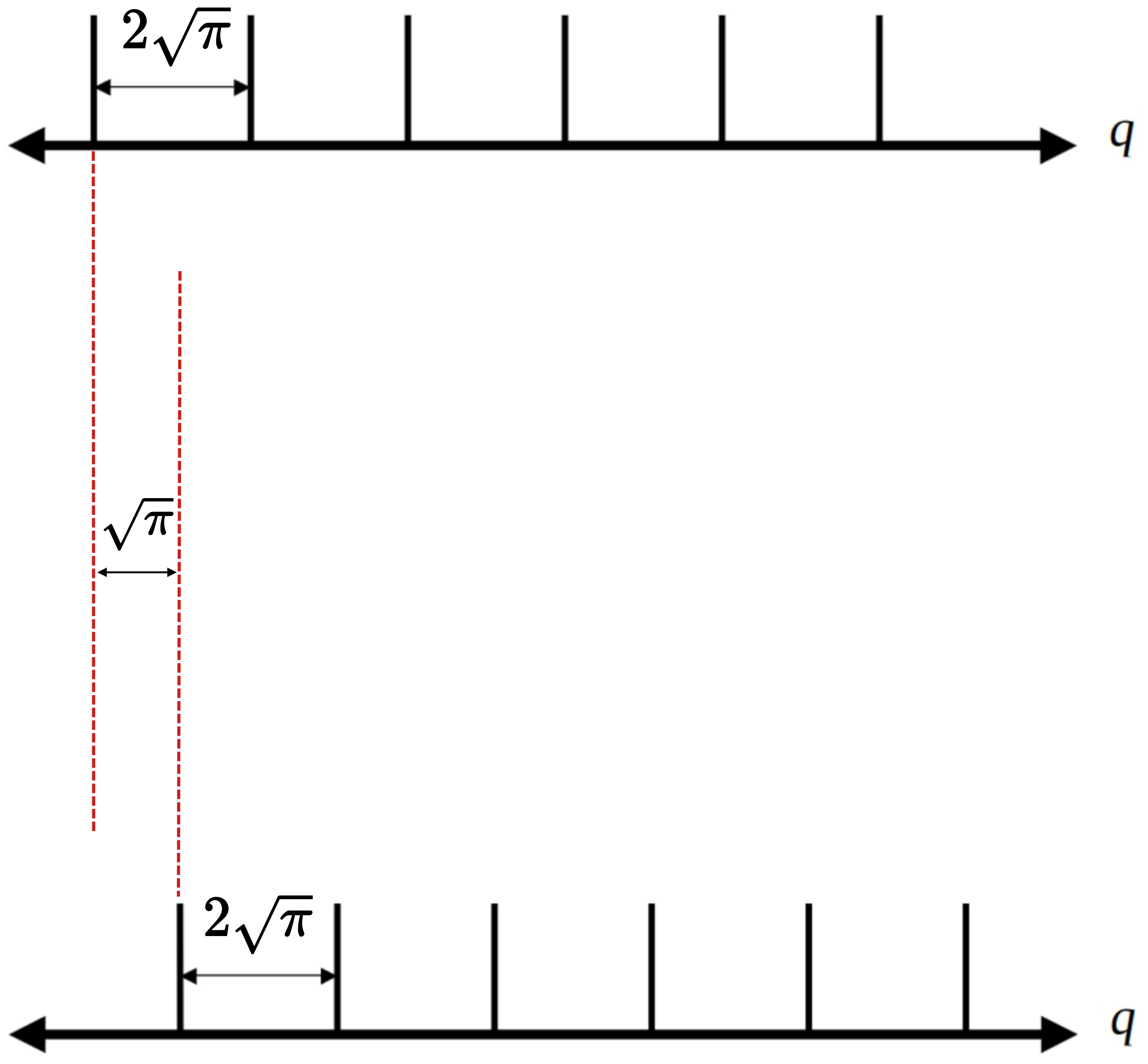
$$Z|j\rangle = (-1)^j|j\rangle$$

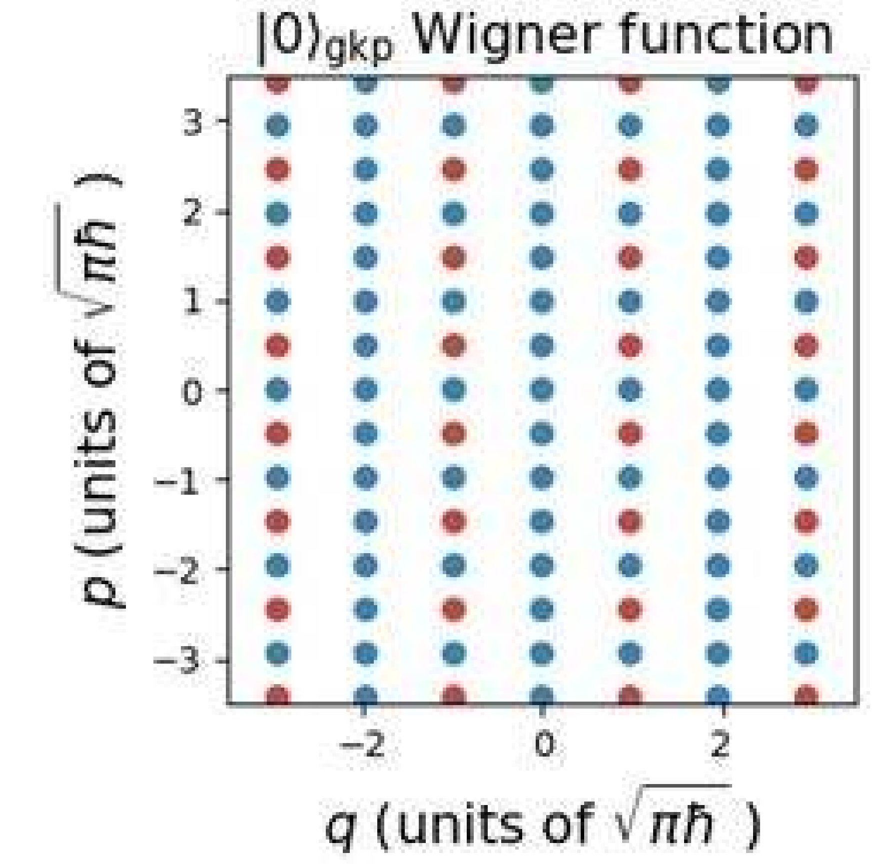
Code space is ***stabilized*** by $\langle Z_1 Z_2, Z_2 Z_3 \rangle$

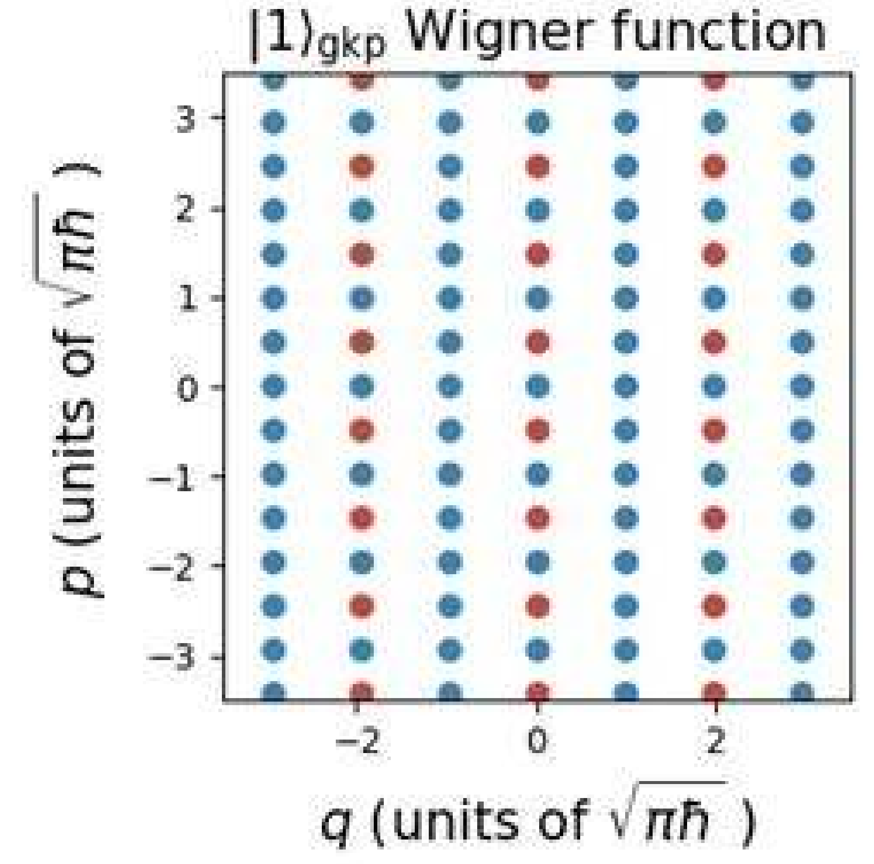
Excursus: symmetries for error correction

$$|0_\mathrm{L}\rangle = \sum_{k=-\infty}^{\infty} \left|2k\sqrt{\pi}\right\rangle_q = \sum_{k=-\infty}^{\infty} \left|k\sqrt{\pi}\right\rangle_p$$

$$|1_\mathrm{L}\rangle = \sum_{k=-\infty}^{\infty} \left|(2k+1)\sqrt{\pi}\right\rangle_q = \sum_{k=-\infty}^{\infty} (-1)^k \left|k\sqrt{\pi}\right\rangle_p$$



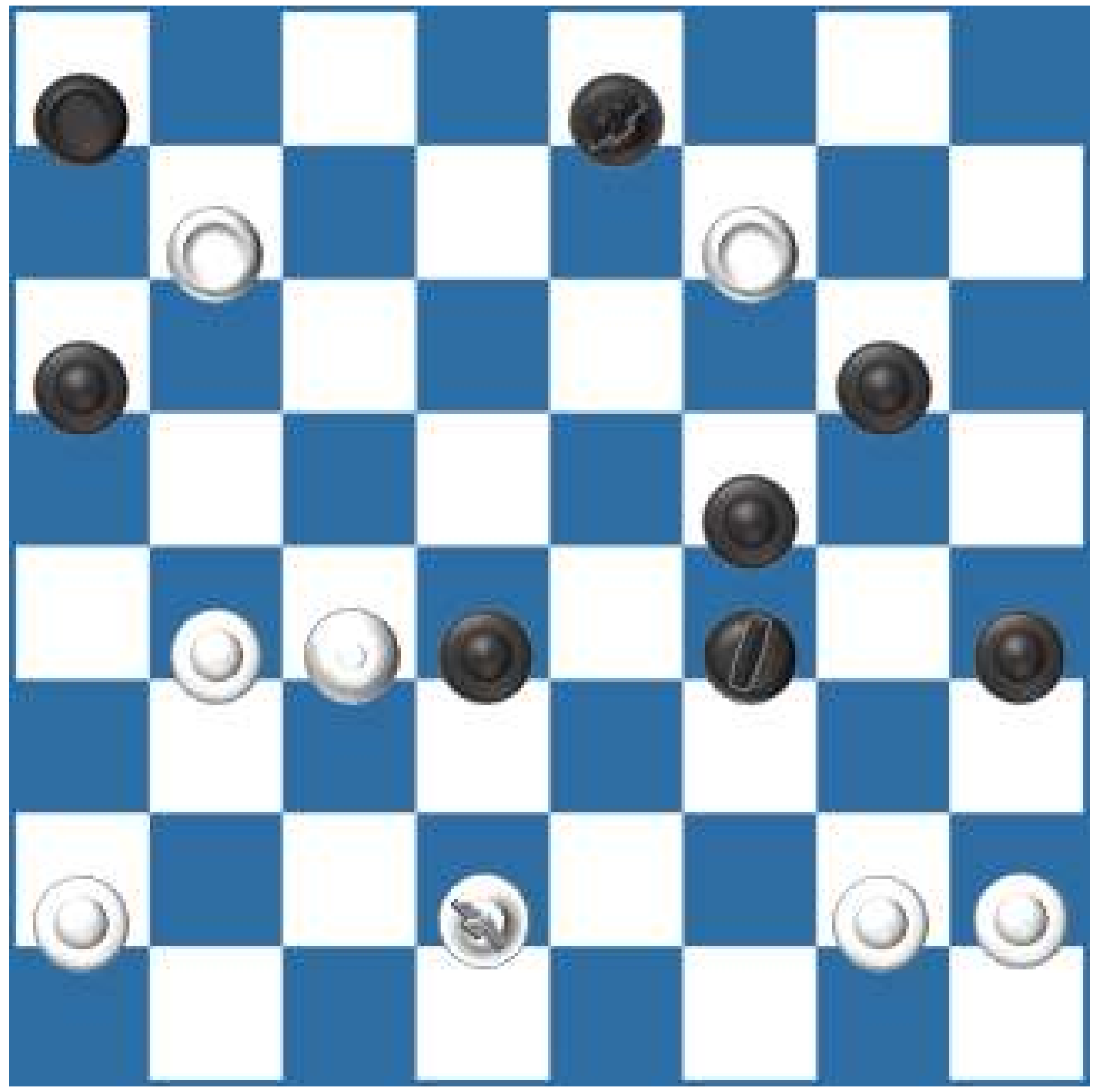




Playing chess on a moving train

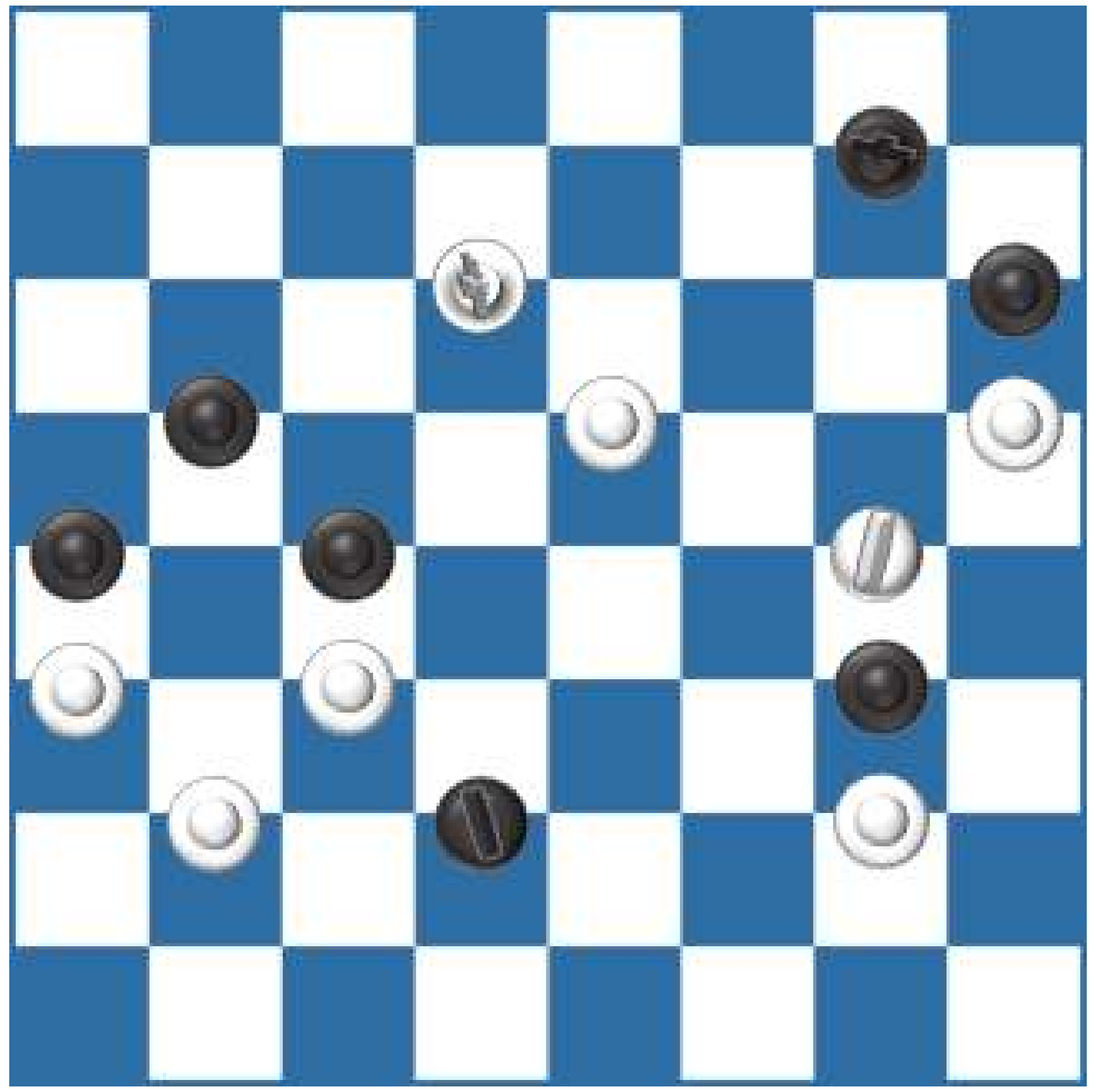
Analogy and figures borrowed from the blog post*: **Riding bosonic qubits towards fault-tolerant quantum computation**, by Ilan Tzitrin, J. Eli Bourassa, and Krishna Kumar Sabapathy

Accumulation of small shifts can result in a shifted position:



Correction: re-center pieces

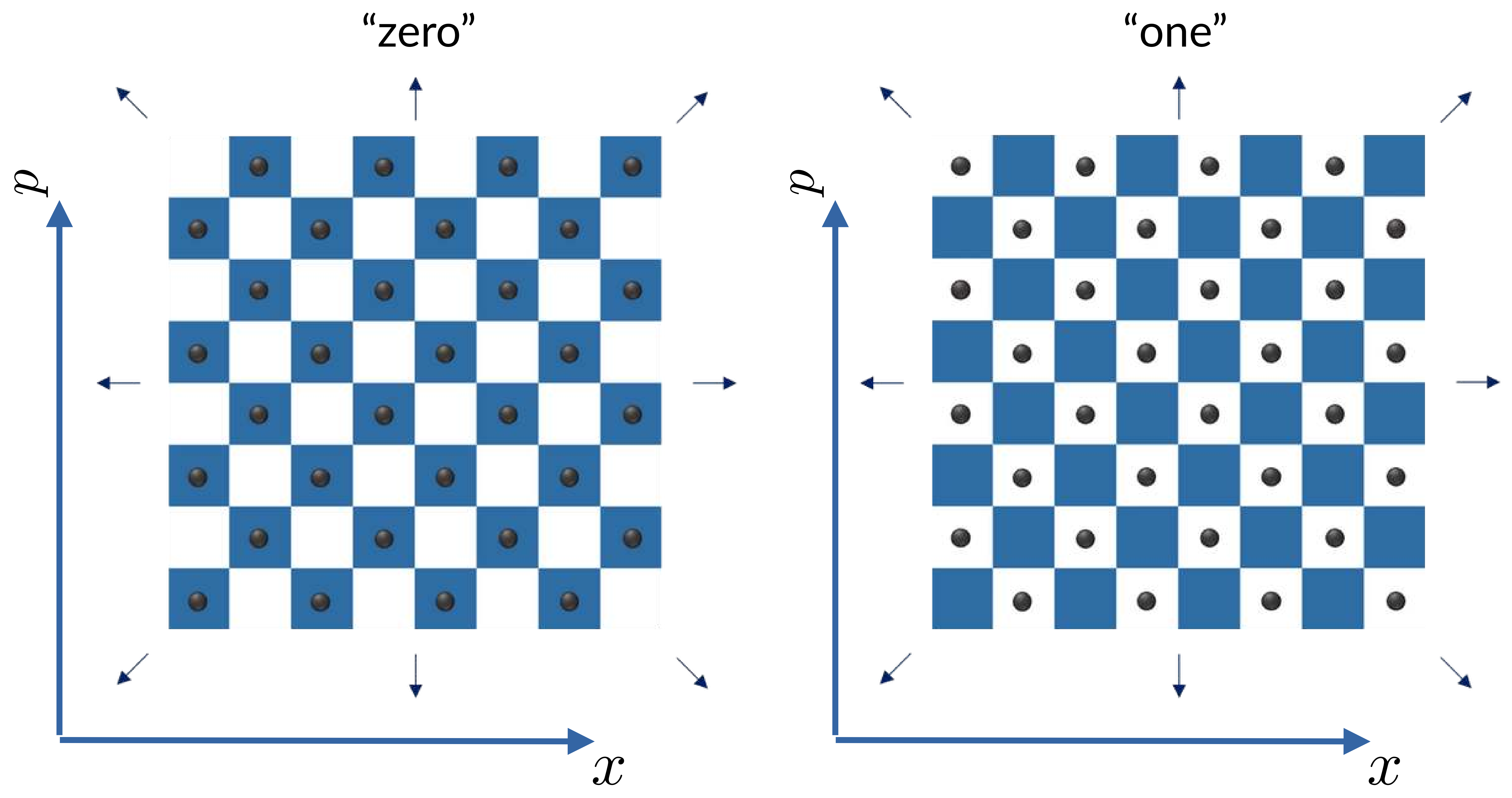
What if the shift is too large?



Correction is ambiguous!

https://www.xanadu.ai/blog/riding-bosonic-qubits-towards-fault-tolerant-quantum-computation

Infinite chessboards



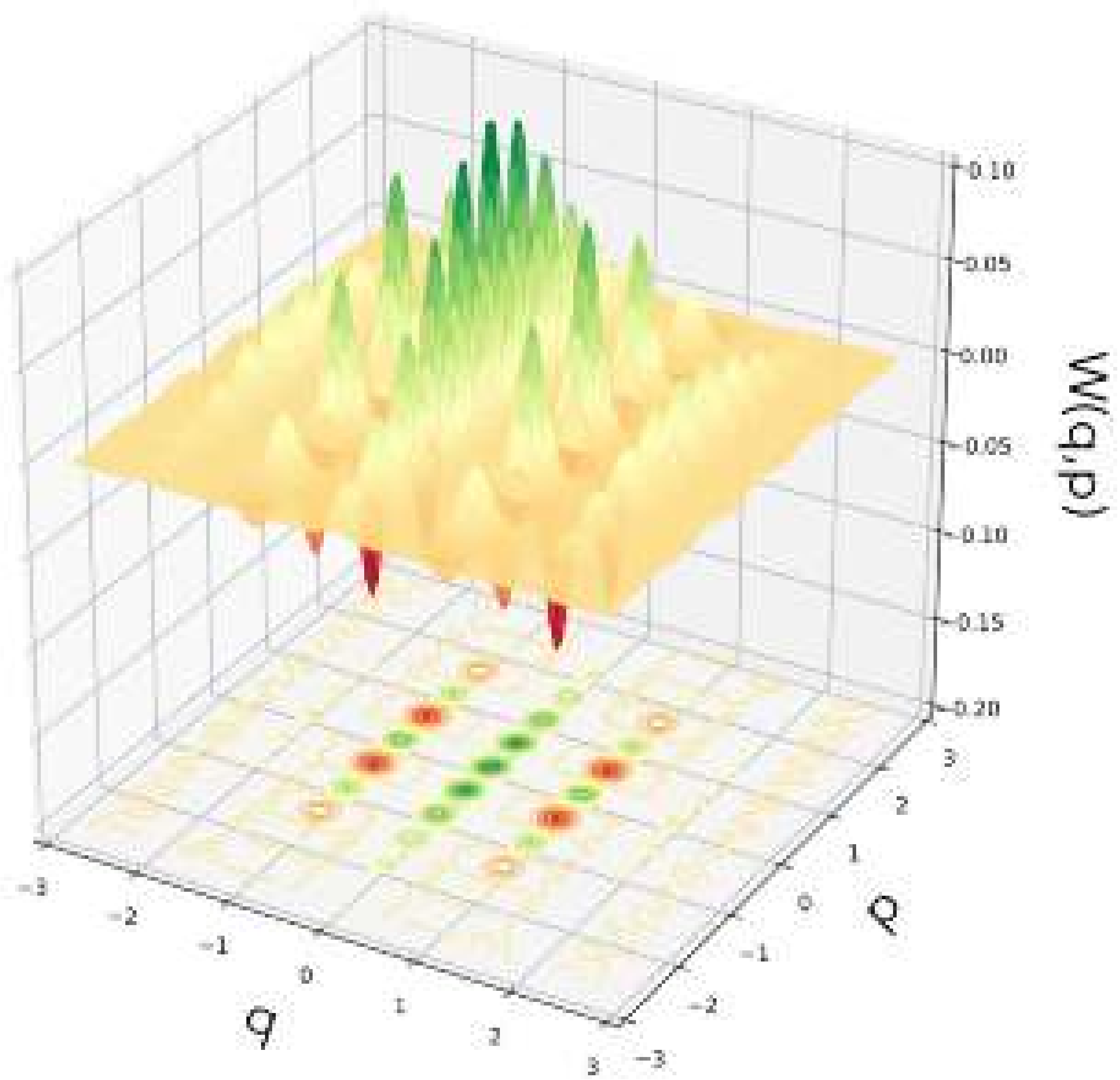
1) Each point has definite "position" and "momentum" but these are not defined for the overall superposition (any compatible result is possible)

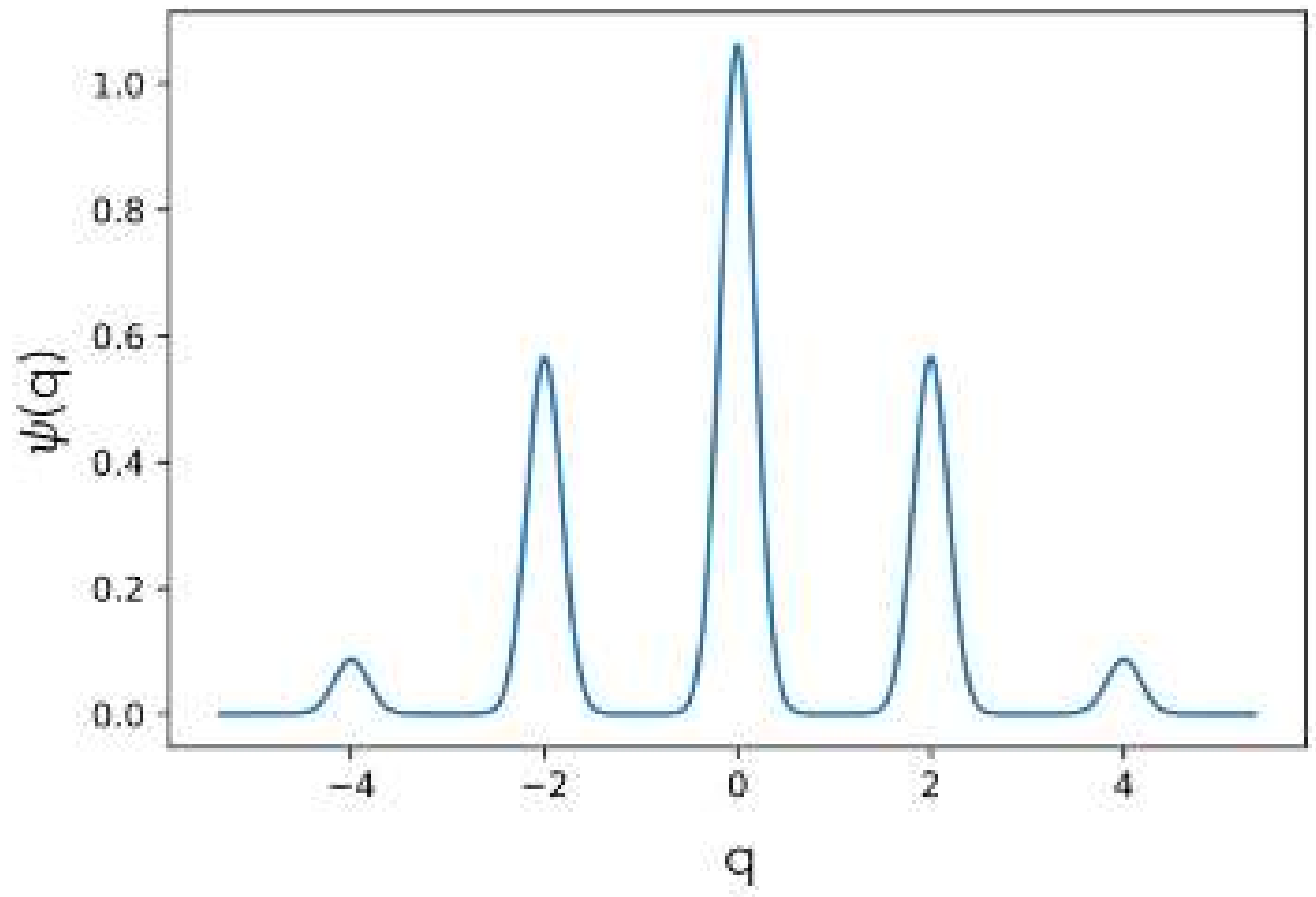
2) Logical states do not overlap

3) Can tolerate "small enough" shifts

https://www.xanadu.ai/blog/riding-bosonic-qubits-towards-fault-tolerant-quantum-computation

Open questions (my work)





1) How do we prepare such states in practice?

2) How do we use more oscillators to tolerate larger shifts?

3) How do we perform (quantum) logic gates?

https://www.xanadu.ai/blog/riding-bosonic-qubits-towards-fault-tolerant-quantum-computation

Open questions (my work)

Quantum
the open journal for quantum science

Gottesman-Kitaev-Preskill codes: A lattice perspective

Jonathan Conrad[1,2], Jens Eisert[1,2], and Francesco Arzani[1]

1) How do we prepare such states in practice?

2) How do we use more oscillators to tolerate larger shifts?

3) How do we perform (quantum) logic gates?