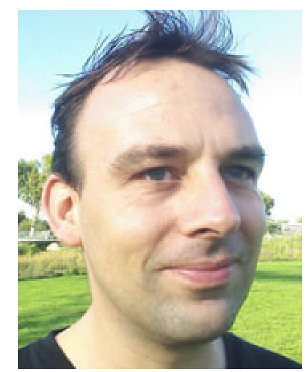
Quantum Secret Sharing with Squeezing and Almost Any Passive Interferometer

F. Arzani, G. Ferrini, F. Grosshans, D. Markham



VanQuTe

Secret Sharing

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\begin{aligned}
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& \quad \oplus \\
b \ \text{(random)} &= 10011 \\
& \quad = \\
c &= 01100
\end{aligned}
$$

- **Access parties**: Groups that can retrieve the secret
- **Adversary structure**: Groups that should not get information
- **Threshold schemes**: any *k* or more players are authorized
- **Quantum** Secret Sharing: secret encoded in a quantum state

Several paradigms

CC: Classical information shared using classical resources

CQ: Classical information shared using quantum resources

→ Improved security

QQ: The secret is a quantum state

Some previous work

- First classical protocol *A. Shamir, Comms of the ACM 22 (11) (1979)*
- First proposal in DV (qubits) *M. Hillery, V. Bužek & A. Berthiaume, PRA 59 (1999)*
  R. Cleve, D. Gottesman & H.-K. Lo, PRL 83 (1999)
- Cluster-state based protocols in DV *D. Markham & B.C. Sanders,PRA 78 (2008)*
- Several proposals in CV... *T. Tyc & B.C. Sanders, PRA 65 (2002)*
  T. Tyc & B.C. Sanders, JoP A 36 (2003)
- ...and experiments *A.M. Lance et al, PRL 92 (2004)*
- CV cluster state - based protocols

P. Van Loock & D. Markham, AIP Conf. Proc. 1363, 256, (2011)

H.-K. Lo & C. Weedbrook, PRA 88 (2013)

So what's new?

~**random encoding**! (...*almost any* passive interferometer)

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Why would I care?

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~**random encoding**! (...*almost any* passive interferometer)

Why would I care?

- Useful to design experiments
- Potentially applicable to share interesting/useful states
- Connections with black holes physics

Continuous Variables

Discrete and Continuous variables

DV : information encoded in *d*-level systems (typically *d* = 2)

$$\alpha\left|0\right\rangle + \beta\left|1\right\rangle$$

$$\Pr\left(0\right) = \left|\alpha\right|^2$$

$$\left|\alpha\right|^2 + \left|\beta\right|^2 = 1$$

$$\mathcal{H} = \mathbb{C}^2$$

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CV : information encoded in observables with continuous spectrum, e.g. : $\hat{q}$, $\hat{p}$

$$\int_{\mathbb{R}} \psi(x)\left|x\right\rangle_q \mathrm{d}x$$

$$\Pr\left(q \in [x, x+\mathrm{d}x]\right) = \left|\psi(x)\right|^2 \mathrm{d}x$$

$$\int_{\mathbb{R}} \left|\psi(x)\right|^2 \mathrm{d}x = 1$$

$$\mathcal{H} = \mathcal{L}^2\left(\mathbb{R}, \mathbb{C}\right)$$

Discrete and Continuous variables

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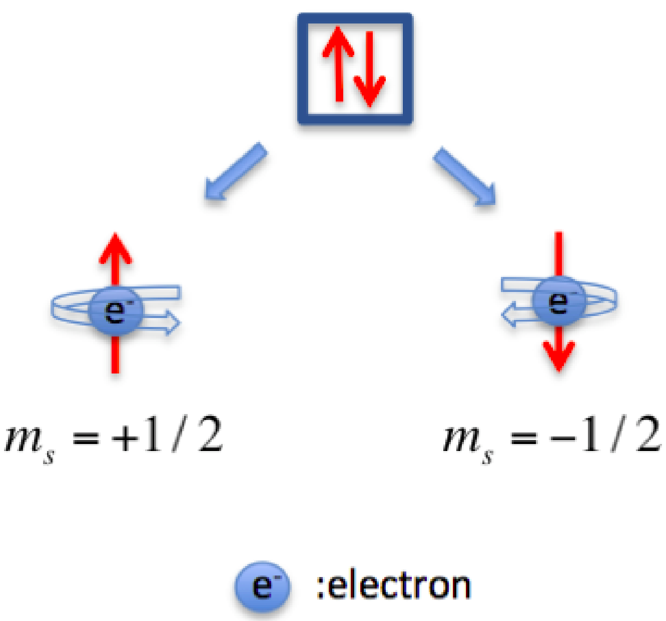
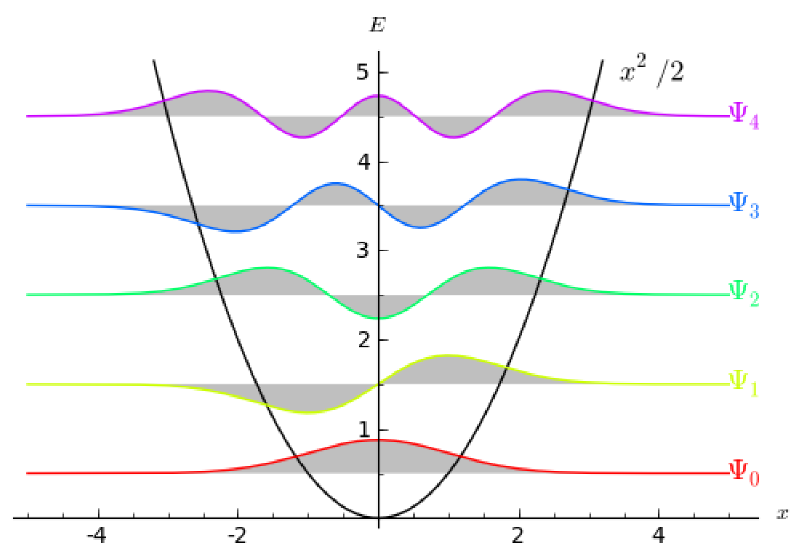
Examples

DV : spins



CV : Harmonic oscillator



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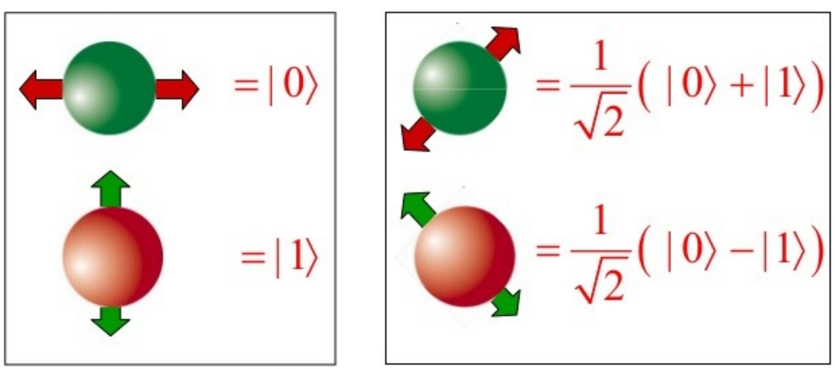
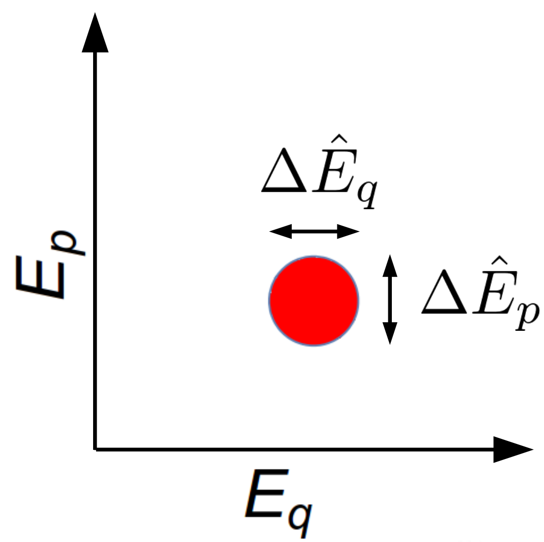
In quantum optics

DV : polarization of single photon



CV : quadratures of the field



$$\hat{E}_q \propto \hat{a} + \hat{a}^\dagger$$
$$\hat{E}_p \propto \hat{a} - \hat{a}^\dagger$$
$$\left[\hat{E}_q, \hat{E}_p\right] = [\hat{q}, \hat{p}]$$

Often simply $\hat{q}$, $\hat{p}$ in the following

Wigner function, Gaussian states & Transformations

CV states can be visualized with a phase-space representation

(Also a useful mathematical tool!)

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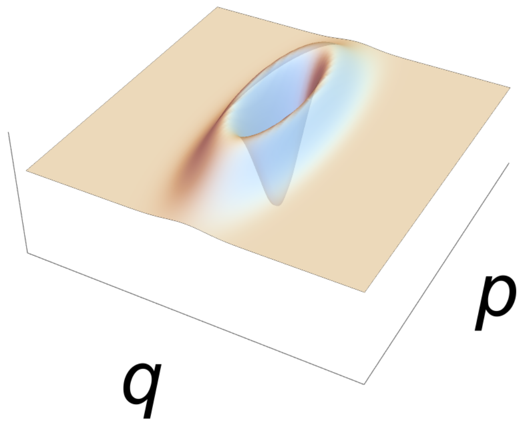
Wigner function ~ Distribution in phase space

$$|\psi\rangle \longrightarrow W_\psi(q,p)$$

$$\int \mathrm{d}p\, W(q,p) = |\langle q\,|\psi\rangle|^2$$

$$\int \mathrm{d}q\, W(q,p) = |\langle p\,|\psi\rangle|^2$$

May be negative!



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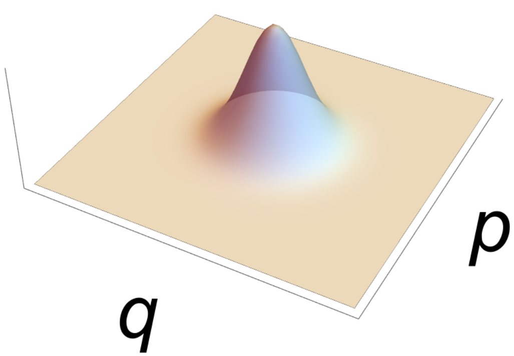
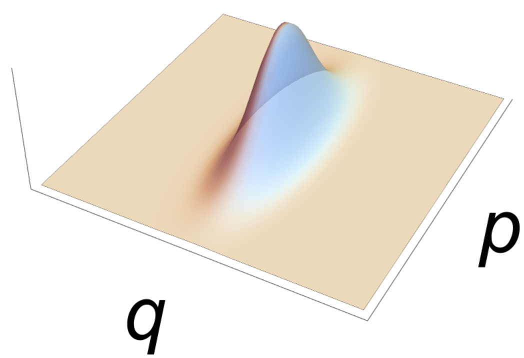


Vacuum → Same marginals



Squeezing

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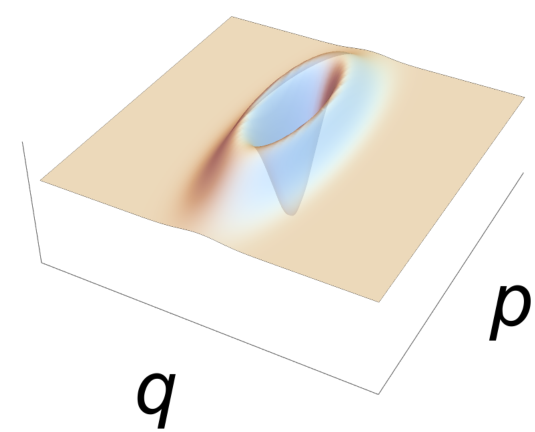

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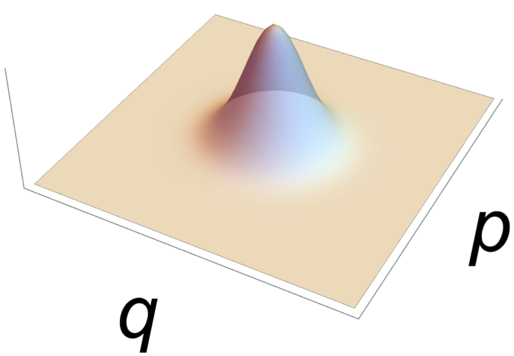

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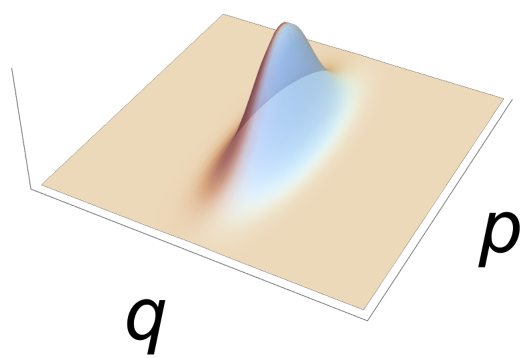

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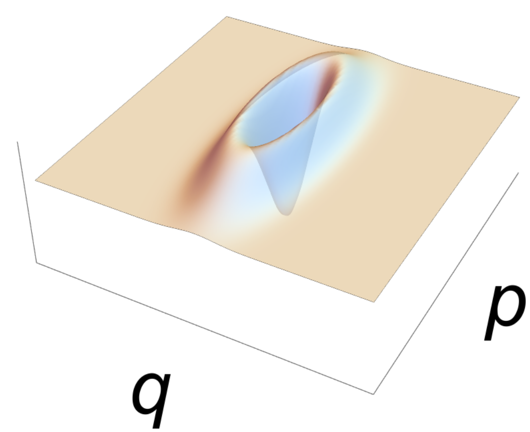
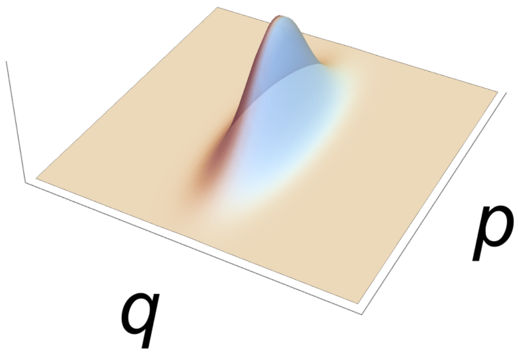


Squeezed states



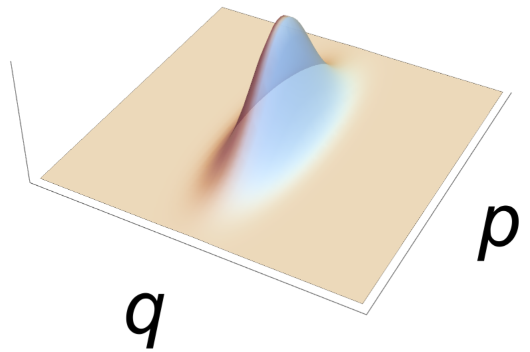
Squeezing

Reduced fluctuations in *q* or *p*

⇓

In the limit, eigen-states of *q* or *p*

Squeezed states



Squeezing

Reduced fluctuations in *q* or *p*

⇩

In the limit, eigen-states of *q* or *p*

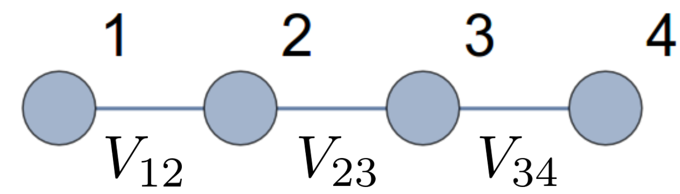
Workhorse of CV Quantum information:

- Easy to produce in the lab (non-linear optical media)
- Deterministic entanglement with passive linear optics
- Used for quantum teleportation
- Experimental production of CV cluster states

Cluster states

$$\exp\left(i\sum_{i>j} V_{ij}\hat{q}_i \otimes \hat{q}_j\right)|0\rangle_p^{\otimes N}$$

- Can be represented as graphs
- Characterized by **nullifier operators**
- Approximated by Gaussian states

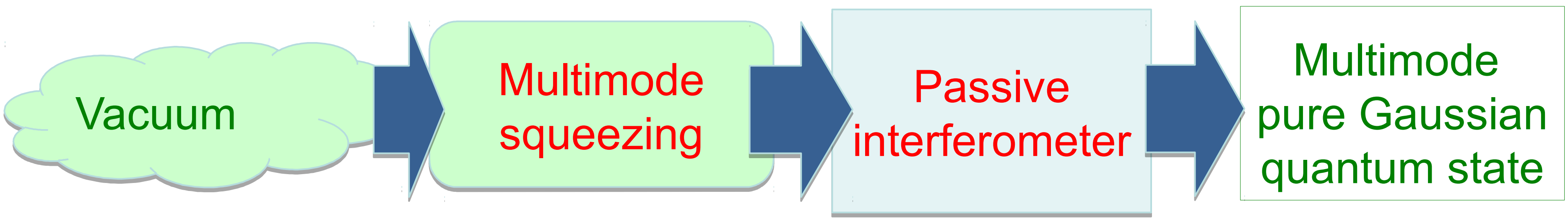


$$\begin{aligned}\hat{\delta}_1 &= \hat{p}_1 - \hat{q}_2\\ \hat{\delta}_2 &= \hat{p}_2 - \hat{q}_1 - \hat{q}_3\\ \hat{\delta}_3 &= \hat{p}_3 - \hat{q}_2 - \hat{q}_4\\ \hat{\delta}_4 &= \hat{p}_4 - \hat{q}_3\end{aligned}$$

Producing Gaussian cluster states

For pure Gaussian states (Quantum Optics):

S. Braunstein,
PRA 71, 055801 (2005)



These operations are **deterministic**!

(No post-selection)

Finite Sqz → Non-zero Q fluctuations → Logical errors

(CV) Quantum secret sharing

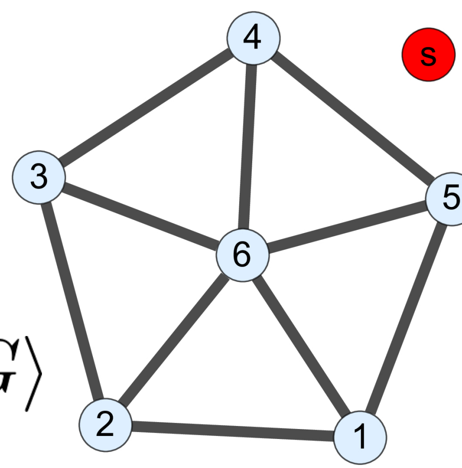
FA, G. Ferrini, F. Grosshans, D. Markham, arXiv:1808.06870

A quantum *(3,5)* scheme with Cluster States

P. Van Loock & D. Markham, AIP Conf. Proc. 1363, 256, (2011)

Start

$|\psi\rangle_s$

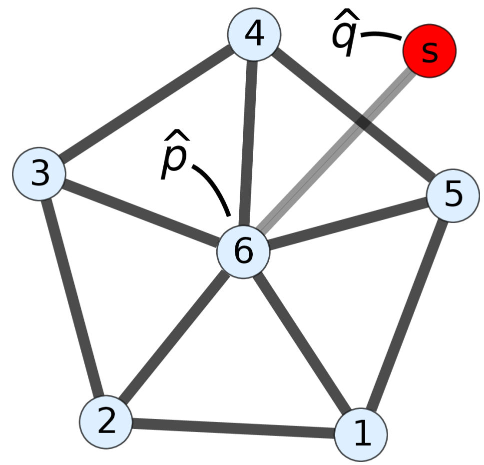


$|G\rangle$

$$\hat{\delta}_j\,|G\rangle = 0 \;\forall j$$

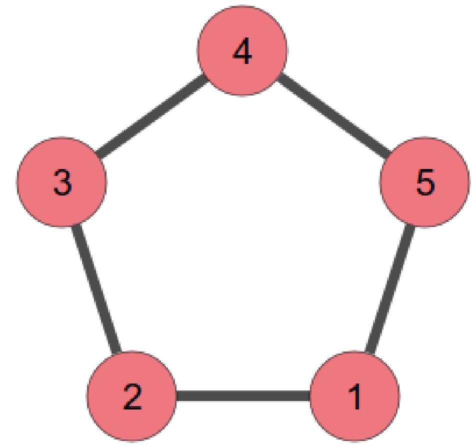
$$|\psi\rangle_s = \int \mathrm{d}y\;\psi(y)\,|y\rangle_{q_s}$$

Teleportation



CV Bell Measurement

Secret is encoded



$$\int \mathrm{d}x\;\psi(x)\,|G(x)\rangle$$

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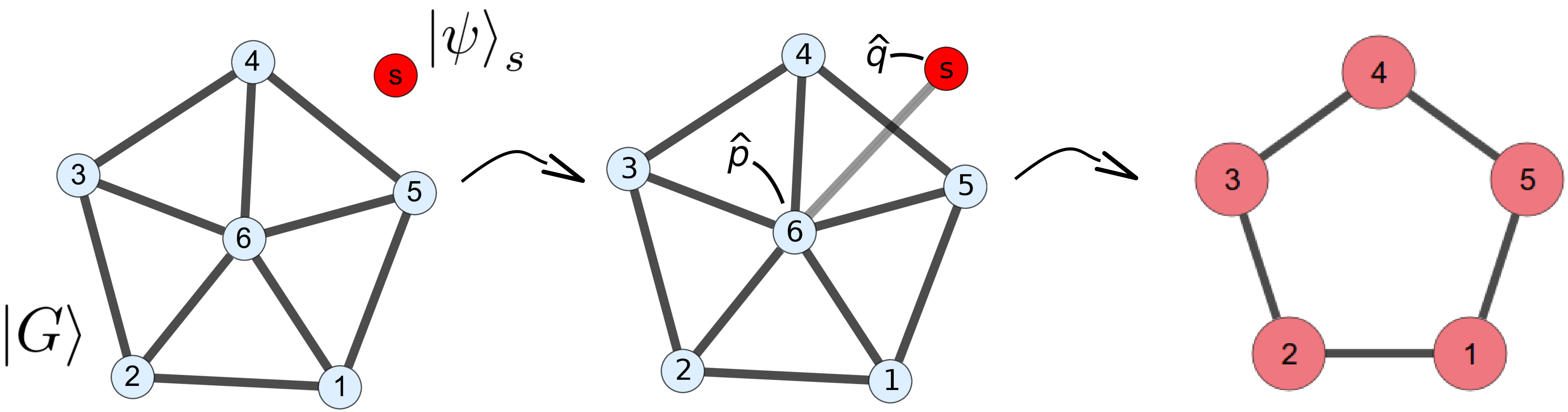

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<u>Logical operators</u>

$$\hat{q}_L = \hat{\delta}_j$$

$$\hat{p}_L = \hat{q}_1 + \hat{q}_2 + \hat{q}_3 + \hat{q}_4 + \hat{q}_5$$

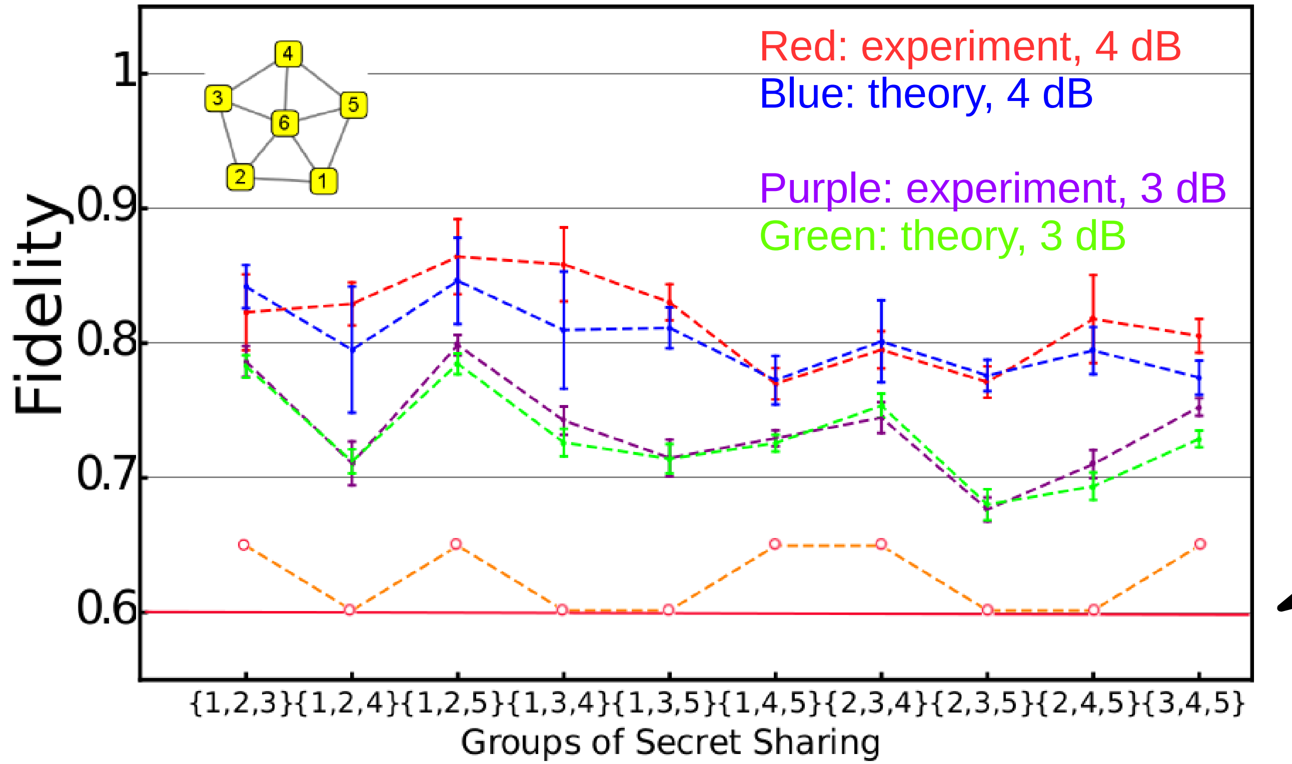
- Same statistics on encoded state as *q*, *p* on secret state
- Can be measured locally by any access party

“Theory inspired” experiment

Y. Cai et al, Nat. Comm. 8, 15645 (2017)

Squeezed states: supermodes

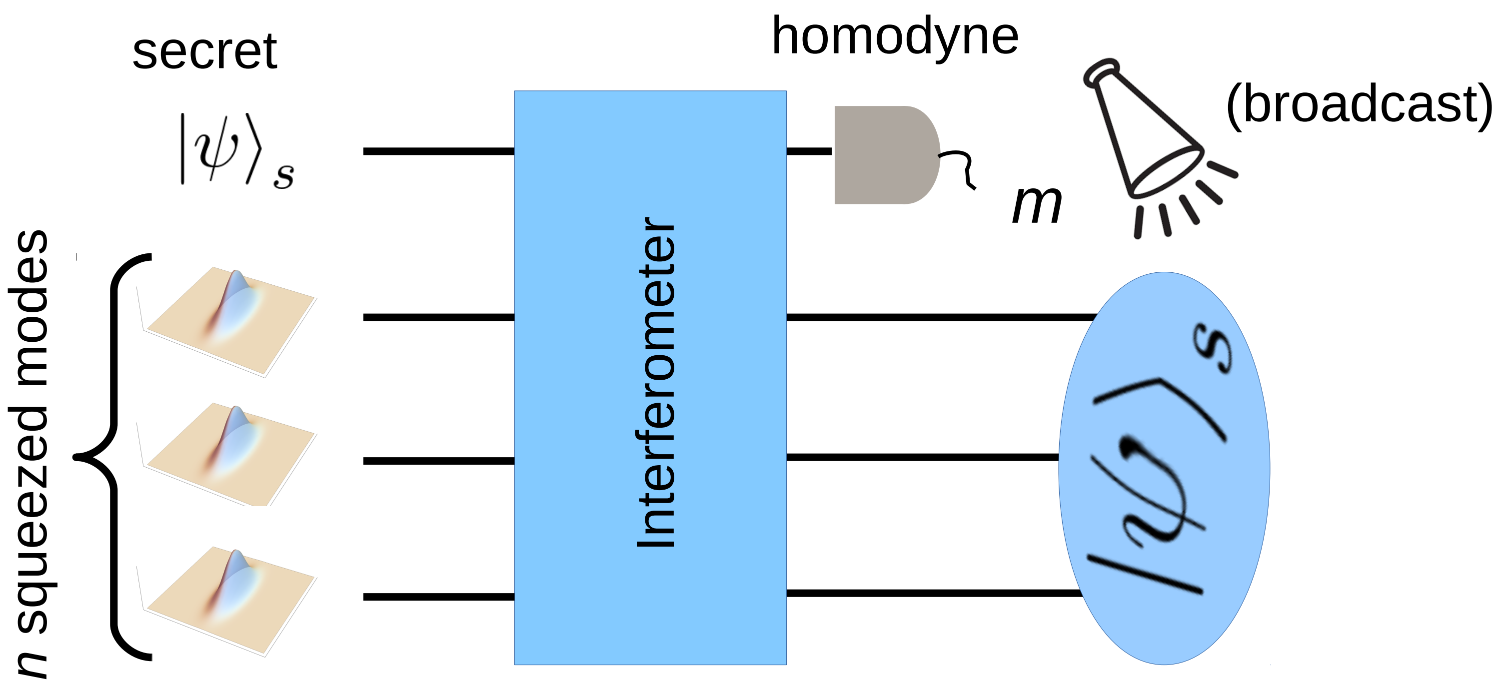
Linear optics → Change of mode basis
(Linear combinations of supermodes)



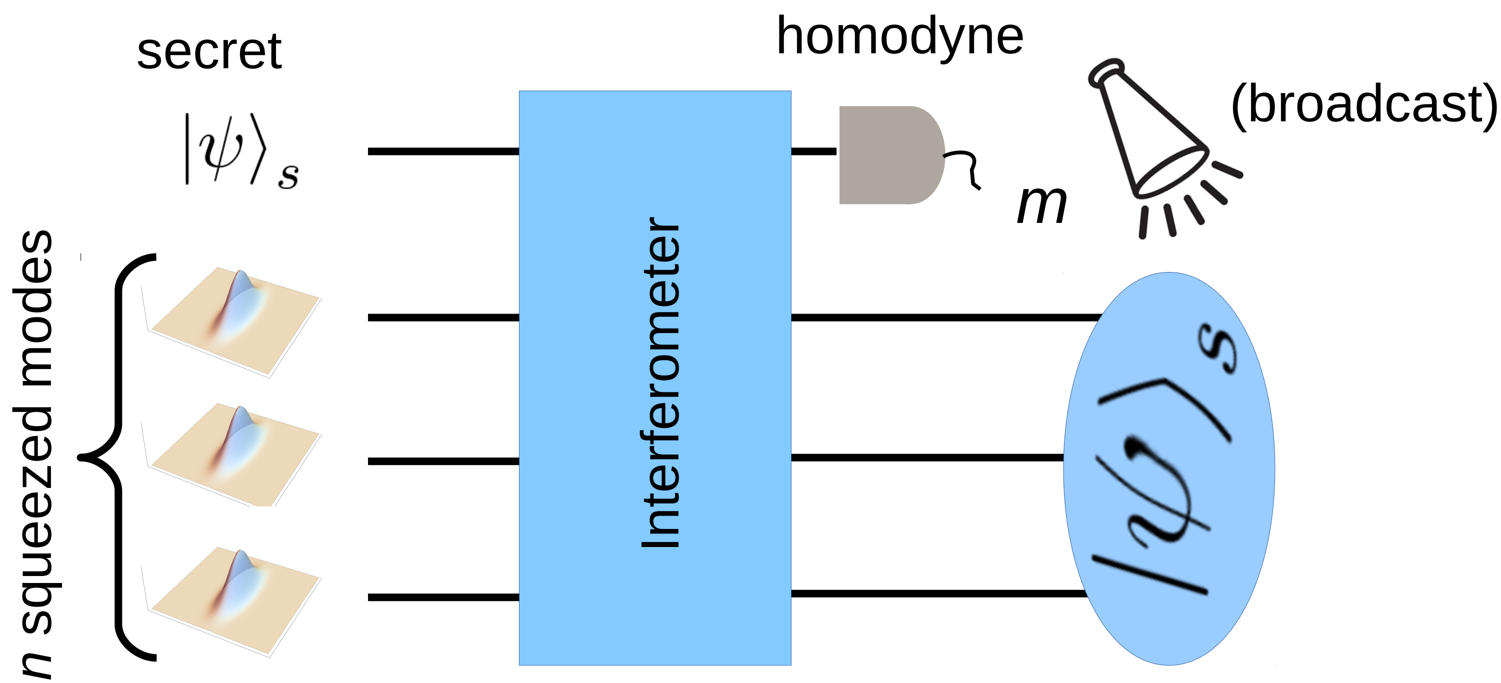
Fidelity above classical bound

The protocol was *simulated*:
modes are not really separated, only gives an estimate of the excess noise

A general CV threshold scheme



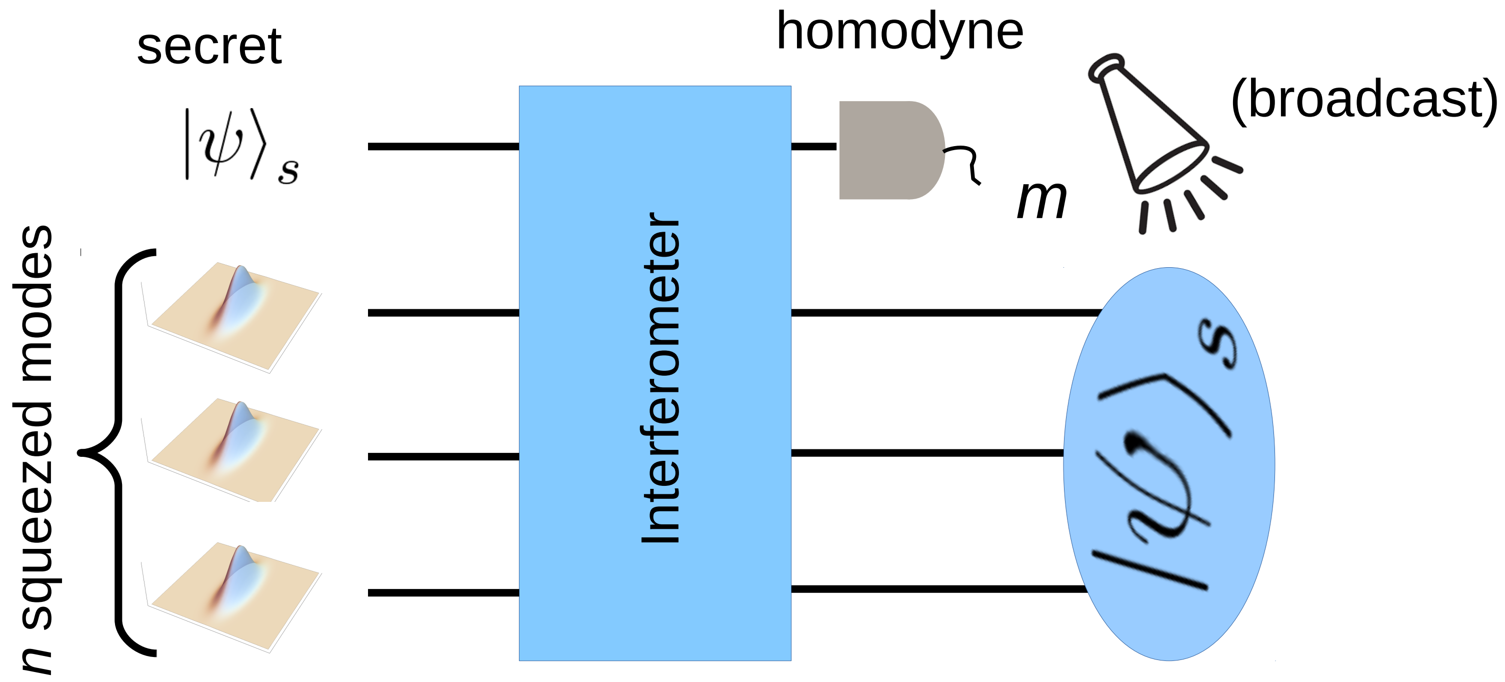
A general scheme



Derived conditions on the interferometer such that each access party can either:

- Measure secret quadratures
- Physically reconstruct the secret

A general scheme



Derived conditions on the interferometer such that each access party can either:

- Measure secret quadratures
- Physically reconstruct the secret

Almost all passive interferometers can be used for Quantum Secret Sharing with squeezed states

(In the sense of Haar measure)

Idea of the proof

Gaussian transformations and Symplectic matrices

$$\boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{p} \end{pmatrix} \qquad [\xi_j, \xi_k] = iJ_{jk} \qquad J = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}$$

Standard symplectic form

Gaussian transformations and Symplectic matrices

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Standard symplectic form

Unitary Gaussian transformations

$$U_G^\dagger \boldsymbol{\xi} U_G = \underbrace{S\boldsymbol{\xi}}_{\text{Phase-space deformation}} + \underbrace{\boldsymbol{x}}_{\text{Phase-space translation}} = \boldsymbol{\xi}'$$

Symplectic Group

$$[\xi'_j, \xi'_k] = iJ_{jk} \iff S^T J S = J$$

$$\mathrm{Sp}\,(2n, \mathbb{R})$$

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Phase-space translation

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$$\mathrm{Sp}\left(2n, \mathbb{R}\right)$$

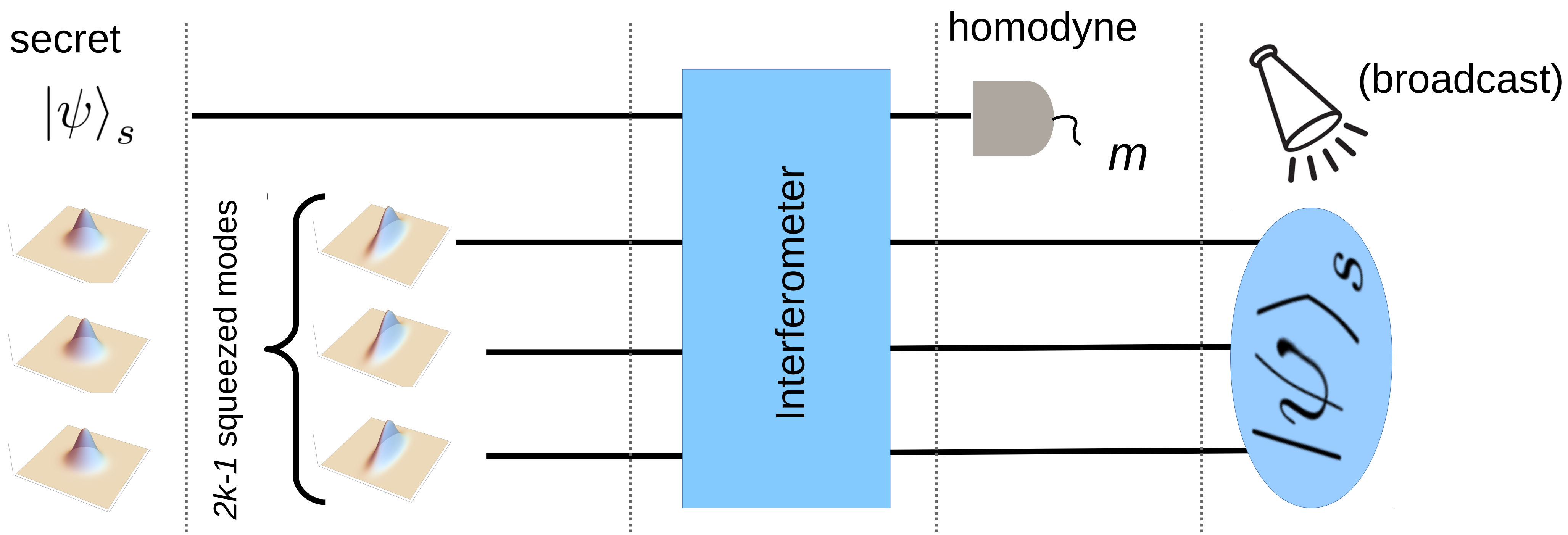
Squeezing:

$$K = \mathrm{diag}\left(e^{r_1}, \ldots, e^{r_n}, e^{-r_1}, \ldots, e^{-r_n}\right)$$

Linear optics (passive **interferometers**):

$$R = \begin{pmatrix} X & -Y \\ Y & X \end{pmatrix}, \qquad X + iY \in U\left(n\right)$$

The scheme revisited



$$\begin{pmatrix} q^s \\ \boldsymbol{q}^{(0)} \\ p^s \\ \boldsymbol{p}^{(0)} \end{pmatrix}$$

$$\boldsymbol{\xi}^{\mathrm{sqz}} = \begin{pmatrix} q^s \\ \boldsymbol{q}^{\mathrm{sqz}} \\ p^s \\ \boldsymbol{p}^{\mathrm{sqz}} \end{pmatrix}$$

$$\begin{pmatrix} q^d \\ \boldsymbol{q}^{\mathrm{net}} \\ p^d \\ \boldsymbol{p}^{\mathrm{net}} \end{pmatrix} = R\boldsymbol{\xi}^{\mathrm{sqz}}$$

$$p^d \mapsto m$$

$$\begin{pmatrix} q_j^{\mathrm{sqz}} \\ p_j^{\mathrm{sqz}} \end{pmatrix} = \begin{pmatrix} e^{r_j} q_j^{(0)} \\ e^{-r_j} p_j^{(0)} \end{pmatrix}$$

$$R = \begin{pmatrix} X & -Y \\ Y & X \end{pmatrix}$$

Decoding conditions

Each player has 2: $$\xi_j^{\mathrm{net}} = \sum_l M_{jl} q_l^{\mathrm{sqz}} + \sum_l N_{jl} p_l^{\mathrm{sqz}} + \alpha_j p^s + \beta_j q^s$$

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noise

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Goal: Get rid of <u>these</u>

Decoding conditions

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Goal: Get rid of these, *solve* for these

Solving linear systems → Submatrices of S_L must be non-singular

Decoding conditions and the Haar measure

$$\begin{cases} Y_{2k,l} \neq 0 \\ \det\left(T^{A}\right) \neq 0 \end{cases}$$

To eliminate the first anti-squeezed *q*

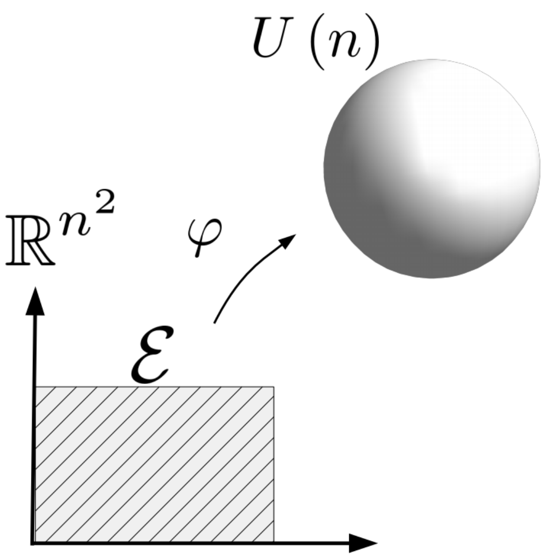
For (all) ***A*** to retrieve the secret

Decoding conditions and the Haar measure

$$\begin{cases} Y_{2k,l} \neq 0 & \text{To eliminate the first anti-squeezed } q \\ \det\left(T^A\right) \neq 0 & \text{For (all) } \boldsymbol{A} \text{ to retrieve the secret} \end{cases}$$

Haar measure = uniform probability measure on *U(n)*

Coefficient of unitary matrices = real analytic functions of “angles”



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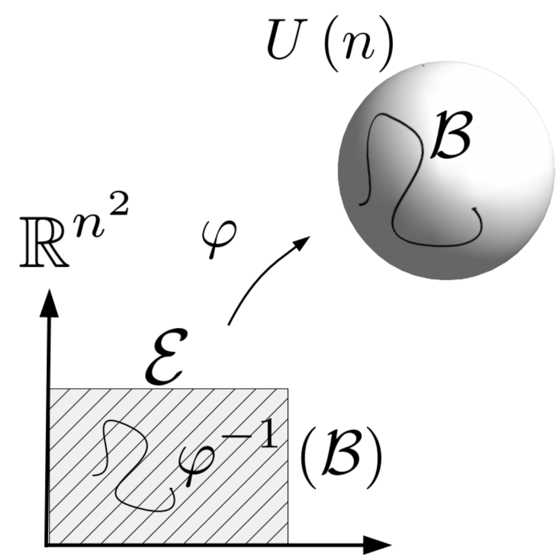
"**= 0**" in corresponds to null set of real analytic functions → zero measure

→ corresponding matrices have zero Haar measure

B. Mityagin, arXiv:1512.07276 (2015)



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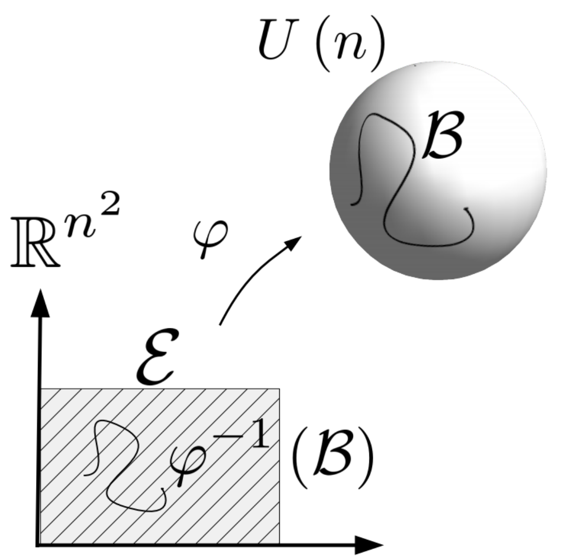
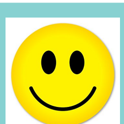


If  : ***A*** can sample

$$q_{\rm s} + \sum_{l=1}^{n-1} B_{1l} p_l^{\rm sqz} = \sum_{j=1}^{j=k} \alpha_j \left(\cos\theta_j Q_j^A + \sin\theta_j P_j^A\right)$$

Or construct a unitary Gaussian decoding

Conclusions & Outlook

- A general scheme with Gaussian resources

  → Works for almost any interferometer

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- Decoding only requires unitary Gaussian transformations

Conclusions & Outlook

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TODO:

- Capacity? (Classical, quantum, private)
- Robust to losses?
- Optimize interferometer?
- Experiments?

Thank you!

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Appendix

Gaussian transformations and Symplectic matrices

$$\boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{p} \end{pmatrix} \qquad [\xi_j, \xi_k] = iJ_{jk} \qquad J = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}$$

Standard symplectic form

Unitary Gaussian transformations

$$U_G^\dagger \boldsymbol{\xi} U_G = S\boldsymbol{\xi} + \boldsymbol{x} = \boldsymbol{\xi}'$$

Phase-space deformation

Phase-space translation

Symplectic Group

$$[\xi_j', \xi_k'] = iJ_{jk} \iff S^T J S = J$$

$$\mathrm{Sp}\left(2n, \mathbb{R}\right)$$

Bloch-Messiah:

$$S = R_1 K R_2$$

Squeezing:

$$K = \mathrm{diag}\left(e^{r_1}, \ldots, e^{r_n}, e^{-r_1}, \ldots, e^{-r_n}\right)$$

Linear optics (passive **interferometers**):

$$R = \begin{pmatrix} X & -Y \\ Y & X \end{pmatrix}, \qquad X + iY \in U\left(n\right)$$

Gaussian **CPTP**:
(Stinespring)

Gaussian unitary with
Gaussian ancillae
+
Partial trace

Encoding procedure

noise

Each player has 2: $$\xi_j^{\mathrm{net}} = \boxed{\sum_l M_{jl} q_l^{\mathrm{sqz}} + \sum_l N_{jl} p_l^{\mathrm{sqz}}} + \alpha_j p^s + \beta_j q^s$$

$$\begin{pmatrix} q_j^{\mathrm{sqz}} \\ p_j^{\mathrm{sqz}} \end{pmatrix} = \begin{pmatrix} e^{r_j} q_j^{(0)} \\ e^{-r_j} p_j^{(0)} \end{pmatrix}$$

Goal: Get rid of these!

After measurement:

$$p^{\mathrm{d}} \mapsto m = \sum_{l=1}^{2k-1} Y_{2k,l} q_l^{\mathrm{sqz}} + \sum_{l=1}^{2k-1} X_{2k,l} p_l^{\mathrm{sqz}} + Y_{2k,2k} q_{\mathrm{s}} + X_{2k,2k} p_{\mathrm{s}}$$

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Set of *k* players ***A*** (access party)

$$2k\left[\ \boldsymbol{\xi}^A = M^A \bar{\boldsymbol{q}}^{\mathrm{sqz}} + N^A \boldsymbol{p}^{\mathrm{sqz}} + \boldsymbol{h}_q^A q^s + \boldsymbol{h}_p^A p^s + \boldsymbol{h}_{\mathrm{d}}^A m\right.$$

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Correct

$$\exists R | R M^A = 0 \longrightarrow R\boldsymbol{\xi}^A = R N^A \boldsymbol{p}^{\mathrm{sqz}} + T^A \begin{pmatrix} q^s \\ p^s \end{pmatrix}$$

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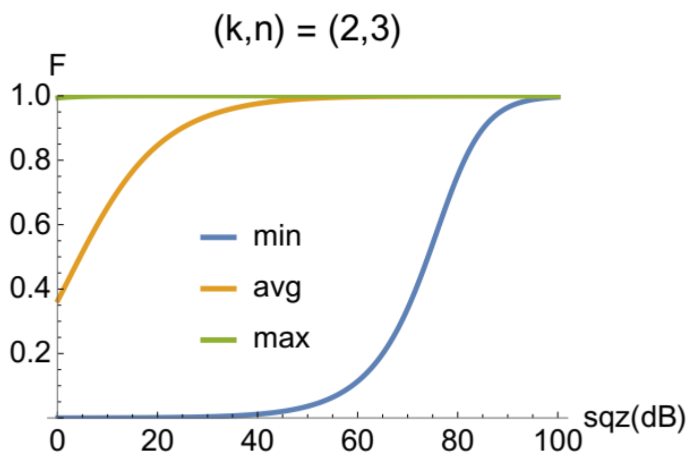
We are done if $\det\left(T^A\right) \neq 0$

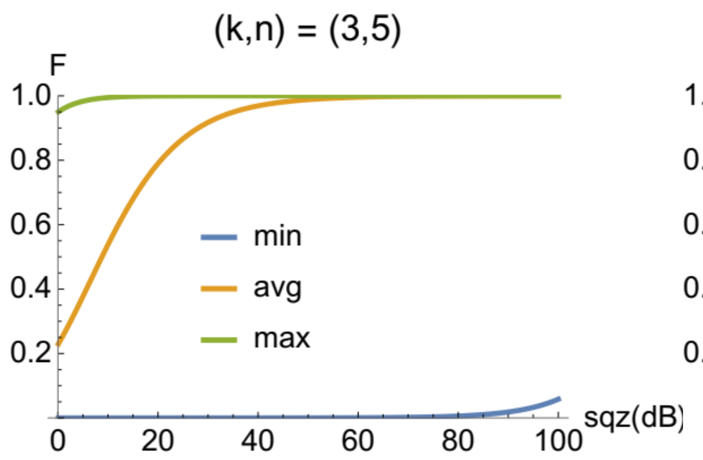
Fidelity vs Squeezing (secret = coherent state)

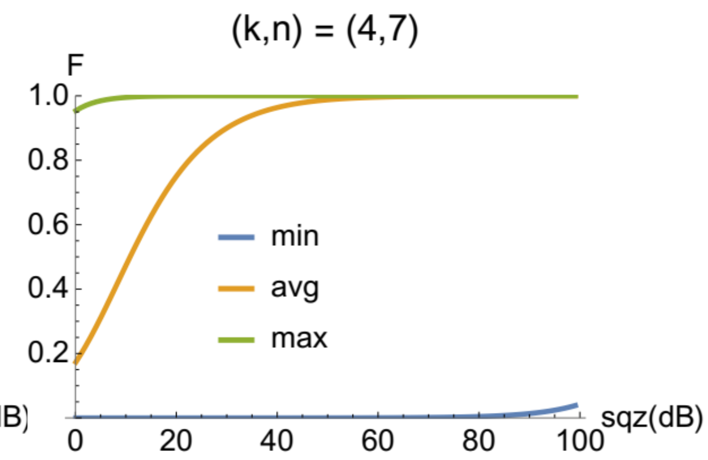
$$\mathcal{F}^A(r) = \left[1 + \sigma^2(r)\,\eta + \sigma^4(r)\,\zeta\right]^{-\frac{1}{2}}$$

$$\Delta^2 p_j^{\mathrm{sqz}} = e^{-2r}/2 \equiv \sigma^2(r)$$

For 1000 randomly generated interferometers







~1 sec ~5 sec ~22 sec

3 APs 10 APs 35 APs

APs = $\binom{n}{k}$