Laboratoire Kastler Brossel
Physique quantique et applications

Measurement Based Quantum Information with Optical Frequency Combs

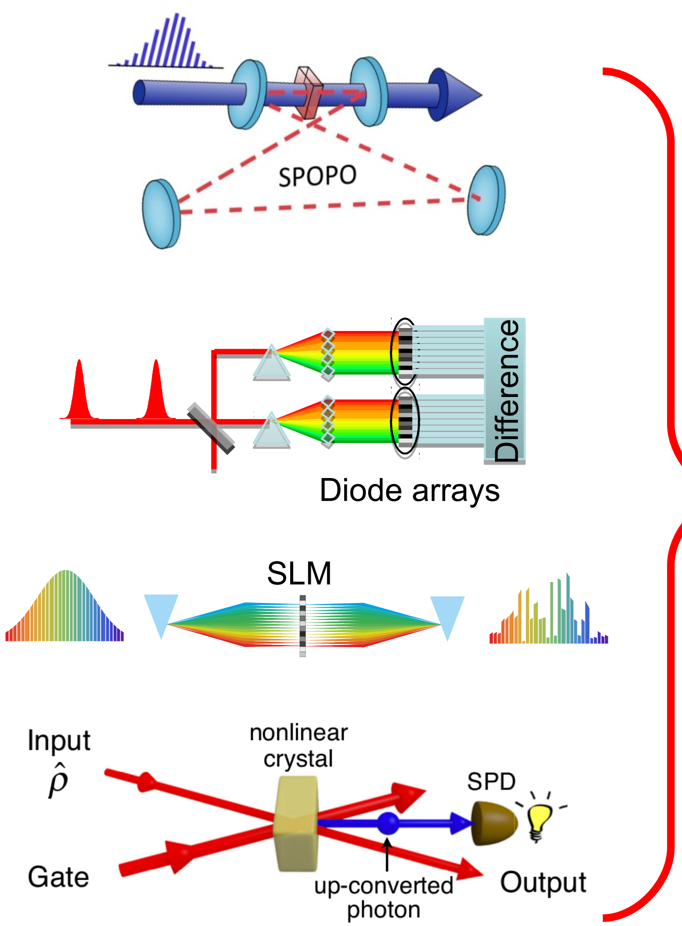
Francesco Arzani

ANR erc QCUMbER cnrs dépasser les frontières

UPMC PARIS UNIVERSITAS ENS ÉCOLE NORMALE SUPÉRIEURE PSL RESEARCH UNIVERSITY

My mission

Devising information processing tasks to perform with available experimental resources, or minimal modifications thereof.



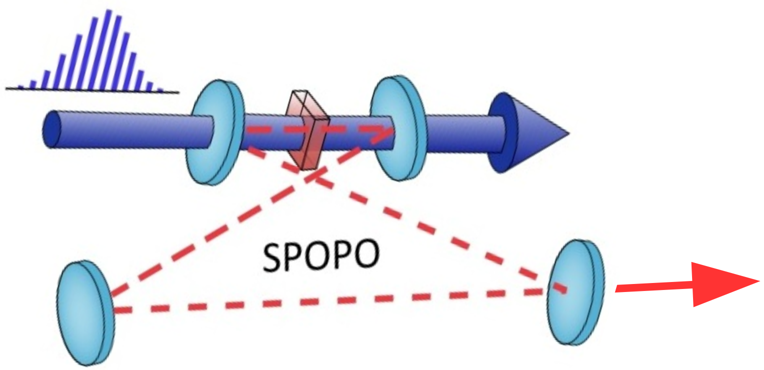
« Resourceful and possessing an encyclopedic knowledge of the physical sciences, he solves complex problems by making things out of ordinary objects, along with his ever-present Swiss Army knife. »

See "MacGyver", Wikipedia

Outline

- Motivation: squeeze many modes to make cluster states
- Multi pixel homodyne detection
- Shape the pump for better clusters
- Introduce non-Gaussianity: subtract photons

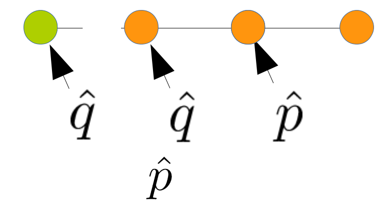
Motivation: CV cluster states



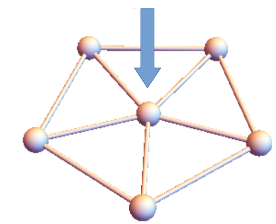
Multi-mode squeezed vacuum

G. Patera et al, EPJD 56, 123-140 (2010)

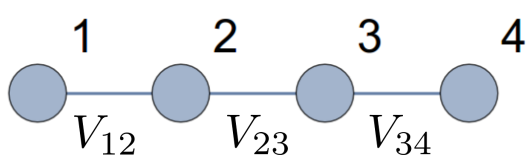
- CV One way QC



- CV Secret Sharing



P. Van Loock, D. Markham, AIP Conf. Proc. 1363, 256, (2011)



$$\begin{aligned}\hat{\delta}_1 &= \hat{p}_1 - \hat{q}_2\\ \hat{\delta}_2 &= \hat{p}_2 - \hat{q}_1 - \hat{q}_3\\ \hat{\delta}_3 &= \hat{p}_3 - \hat{q}_2 - \hat{q}_4\\ \hat{\delta}_4 &= \hat{p}_4 - \hat{q}_3\end{aligned}$$

$$\exp\left(i\sum_{i>j} V_{ij}\hat{q}_i \otimes \hat{q}_j\right)|0\rangle_p^{\otimes N}$$

- Can be represented as graphs
- Characterized by **nullifier operators**
- Approximated by Gaussian states

Vacuum → Multimode squeezing → Mode basis change → Multimode pure Gaussian quantum state

S. Braunstein, PRA 71, 055801 (2005)

What exactly is a mode?

A normalized solution $f_i(\mathbf{r},t)$ of Maxwell's equations

MODE BASIS

$$\int f_n(\vec{r},t)\cdot f_m^*(\vec{r},t)d^3r dt = \delta_n^m$$

OPERATOR BASIS

$$[\hat{a_n}, \hat{a_m}^\dagger] = \delta_n^m$$

Change of modes ⟷ **Linear optics**

$$\hat{E}^+(\vec{r},t) = E_0 \sum_n \hat{a}_n f_n(\vec{r},t)$$

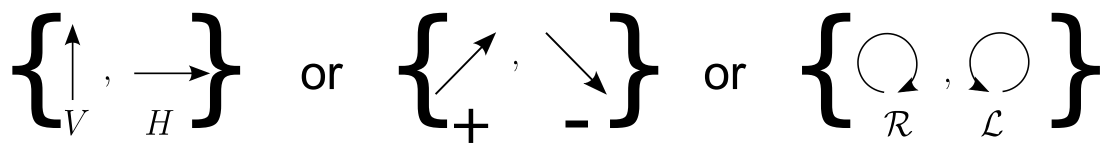
TWO HILBERT SPACES

$$\{\hat{a}_n, f_n(\vec{r},t)\}$$

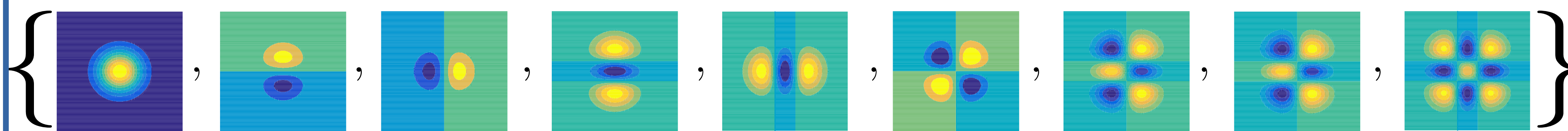
$$|\Psi\rangle = \sum_{n1,n2,\ldots} A_{n1n2\ldots} |n1 : f_1\rangle \otimes |n2 : f_2\rangle \otimes \ldots$$

What exactly is a mode?

Polarization modes



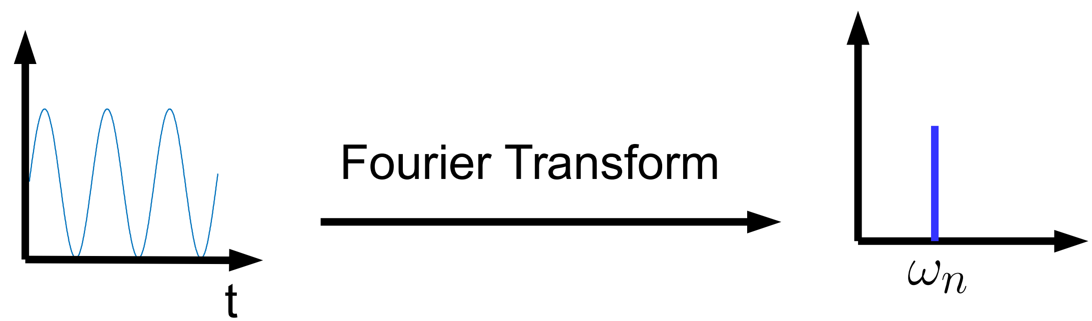
Spatial modes, TEMs



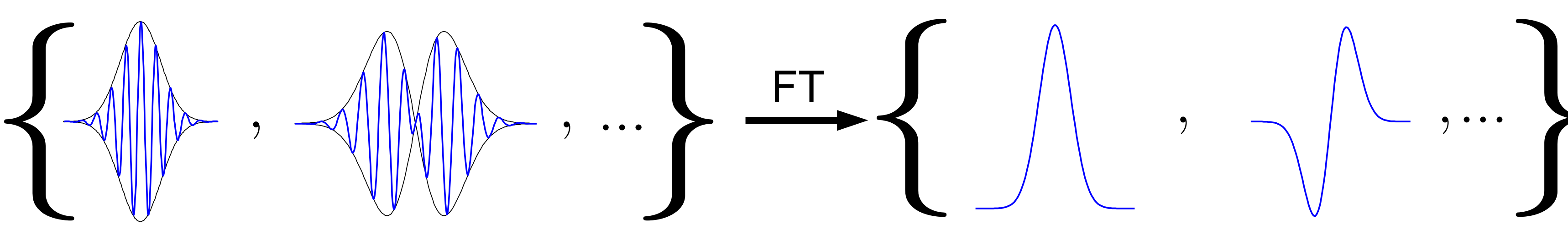
Temporal modes

The ones I will talk about !

Plane waves $\left\{ e^{i\omega_n t} \right\}$



HG modes



(Quantum) Computing and continuous variables

Discrete encoding $\vec{x} = 1001111010101001\ldots$

bits $\vec{x} \mapsto b(\vec{x})$ For any boolean function

qubits $|\vec{x}\rangle|\phi\rangle \mapsto U_b|\vec{x}\rangle|\phi\rangle = |\vec{x}\rangle|b(\vec{x})\rangle$

qu**modes** $|\psi\rangle \in \left(\mathcal{L}^2(\mathbb{R},\mathbb{C})\right)^{\otimes n} \mapsto e^{-itH(\hat{\vec{a}},\hat{\vec{a}}^\dagger)}|\psi\rangle$

General polynomial

Computation with arbitrary encoding

Universal set:

Cluster states + Homodyne

$$\left\{e^{i\hat{q}s}, e^{i\hat{q}^2 s}, e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, e^{i\hat{q}_1\otimes\hat{q}_2}, e^{i\hat{q}^3 s}\right\}$$

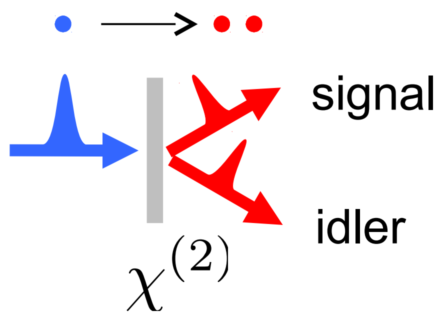
Single-mode, Gaussian

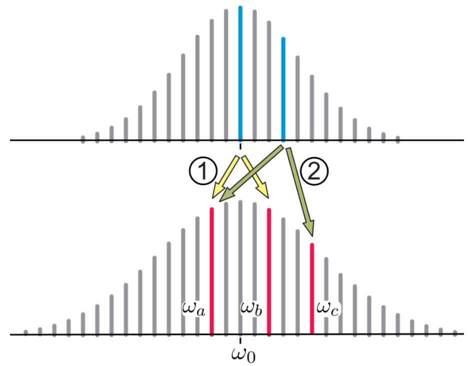
Two-modes C_Z

Non-Gaussian ⟵ Ancillae, Measurement, ...

S. Lloyd and S. L. Braunstein
PRL 82, 1784 (1999)

Multimode squeezing: Parametric Interaction





Interaction Hamiltonian

$$H = i\sum_{m,q} \mathcal{L}_{m,q}\hat{a}^{\dagger}_{\omega_m}\hat{a}^{\dagger}_{\omega_q} + \text{h.c.}$$

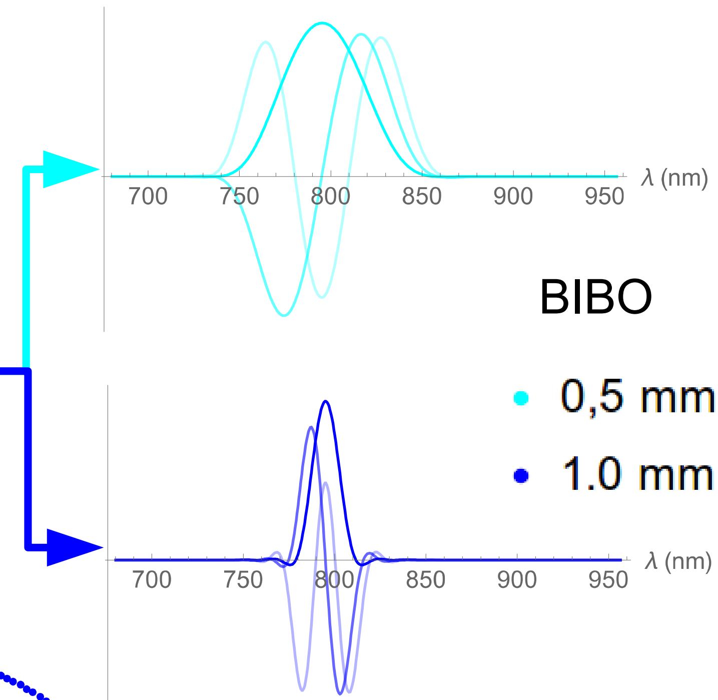
$$\mathcal{L}_{m,q} = \underbrace{\operatorname{sinc}\left(\Delta k_{m,q}\frac{l}{2}\right)}_{\text{Crystal}} \times \underbrace{\alpha\left(\omega_m + \omega_q\right)}_{\text{Pump}}$$

$$U\mathcal{L}U^T = \Lambda$$

Symmetric SVD

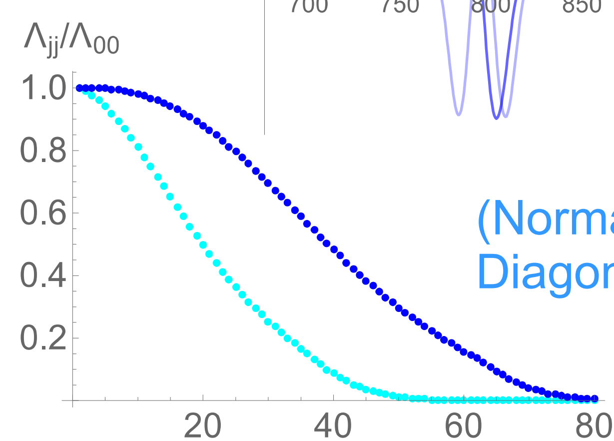
Rows of U

Supermodes

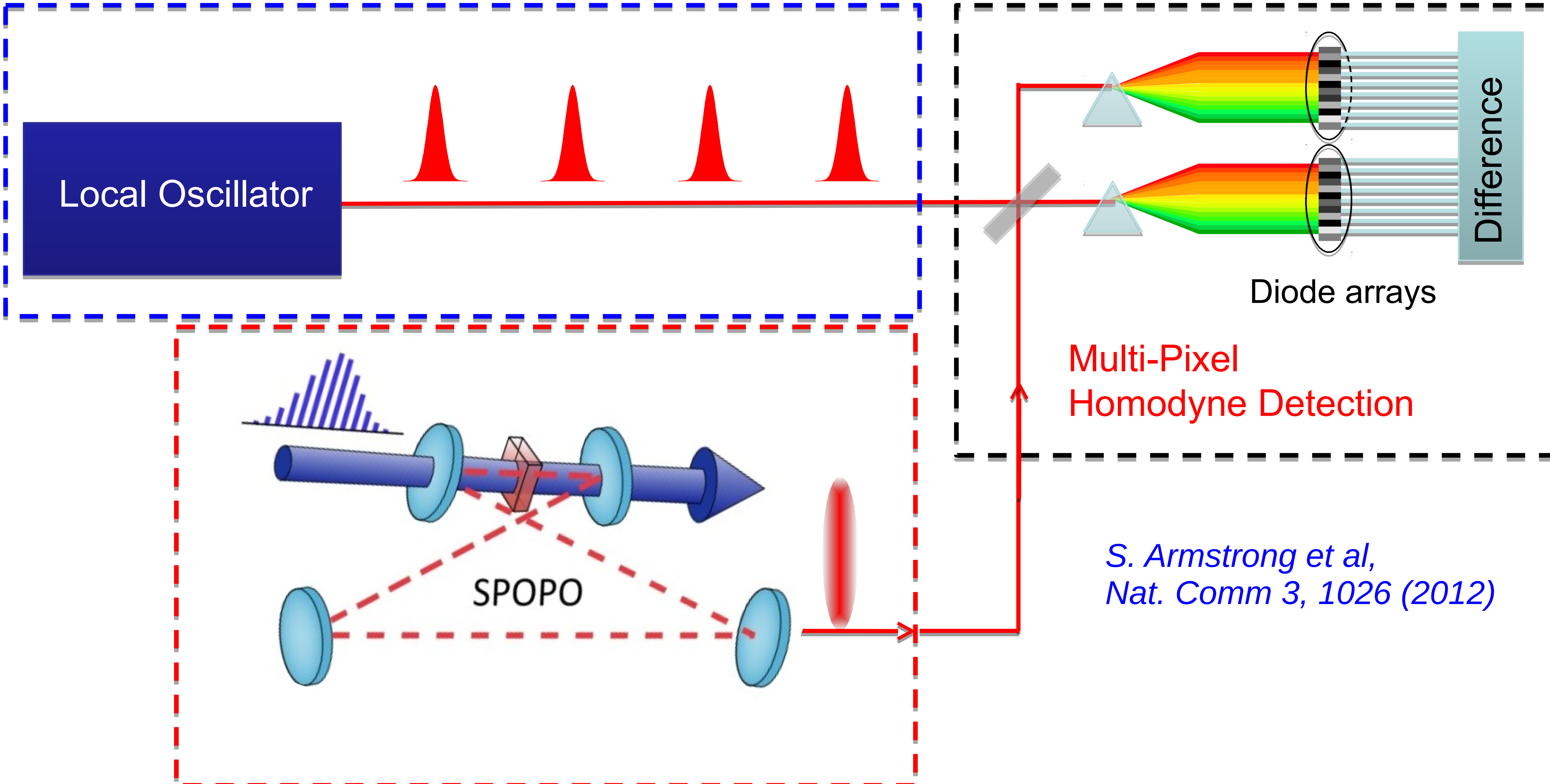


$$H = i\sum_{j} \Lambda_{jj}\left(\hat{S}^{\dagger}_{j}\right)^2 + \text{h.c.}$$

Change of mode basis ? Homodyne !



Multi-Pixel Homodyne Detection



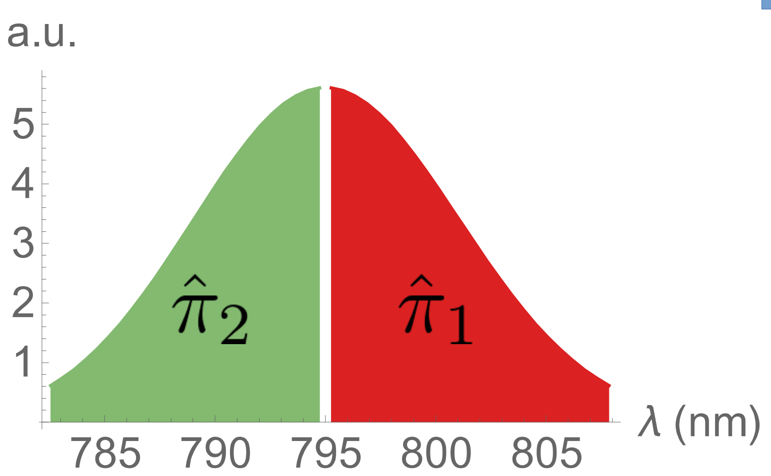
S. Armstrong et al,
Nat. Comm 3, 1026 (2012)

- Modes can be separated easily
- Measurement of one mode does not destroy the rest of the system

Change of modes

Pixels are linear combinations of single frequencies / squeezed modes

$$\hat{\pi}_j = \sum_n \eta_{j,n} \hat{a}_{\omega_n} = \sum_l \zeta_{j,l} \hat{S}_l$$



G. Ferrini et al,
NJP 15, pp.093015 (2013)

➡ **Effective mode-basis change**

Measurement in pixel basis

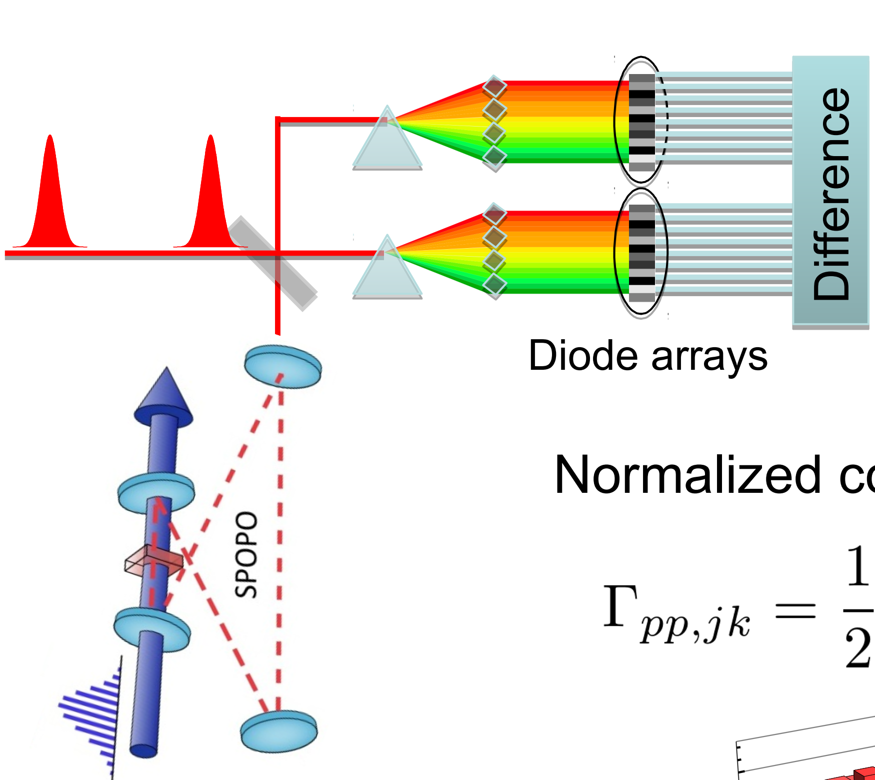
↕

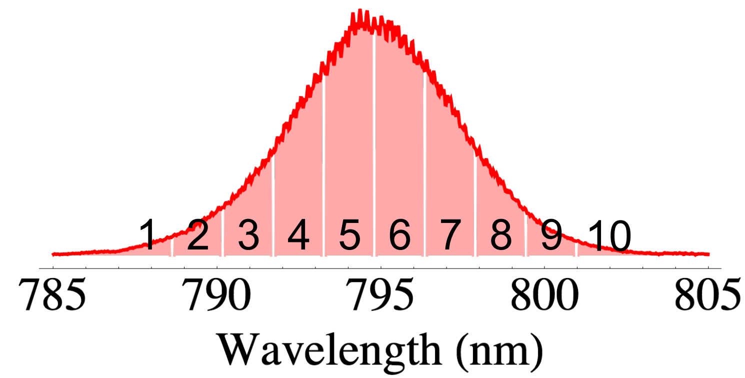
Change of modes and then measure

↕

Linear optics and then measure

State of the System: Covariance Matrix



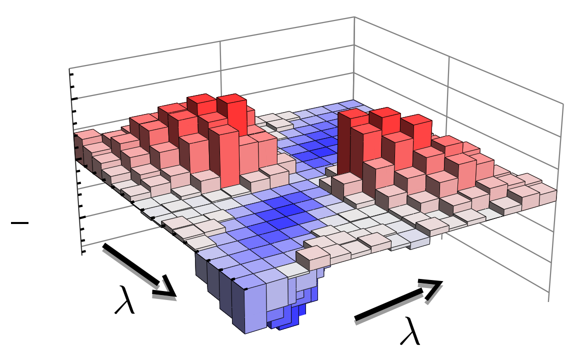


Normalized correlations between frequency bands

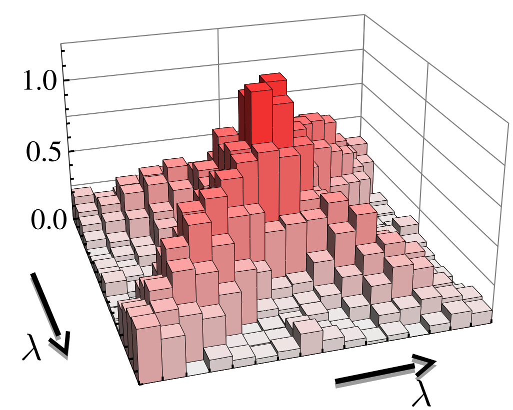
$$\Gamma_{pp,jk} = \frac{1}{2}\langle\{\hat{p}_j, \hat{p}_k\}\rangle$$

$$\Gamma_{qq,jk} = \frac{1}{2}\langle\{\hat{q}_j, \hat{q}_k\}\rangle$$

$$\Gamma_{qp,jk} = \frac{1}{2}\langle\{\hat{q}_j, \hat{p}_k\}\rangle$$



16-mode Covariance matrix of Phase Quadrature



Amplitude Quadrature

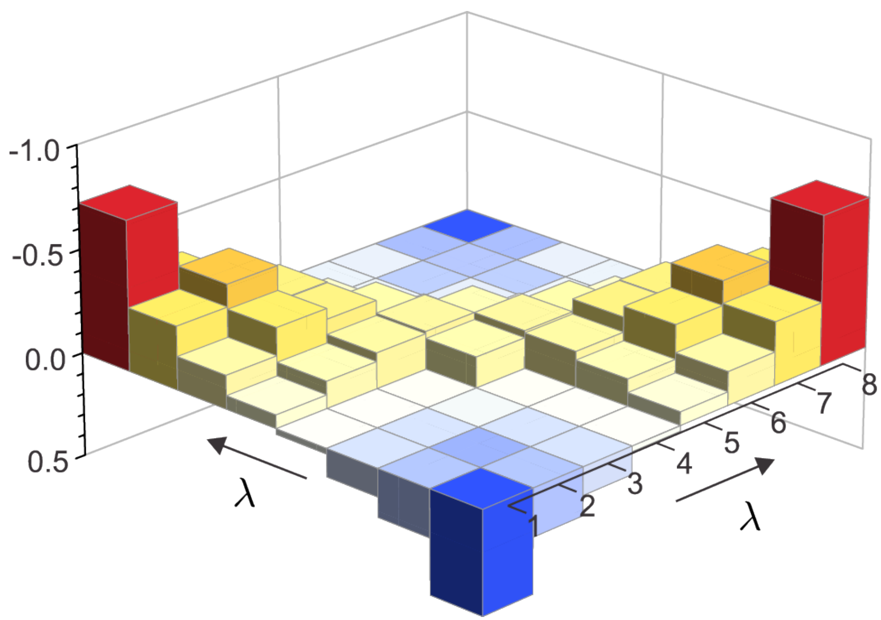
Highly entangled state!

S. Gerke et al, PRL 114, 050501 (2015)

Reconstructing the squeezed modes

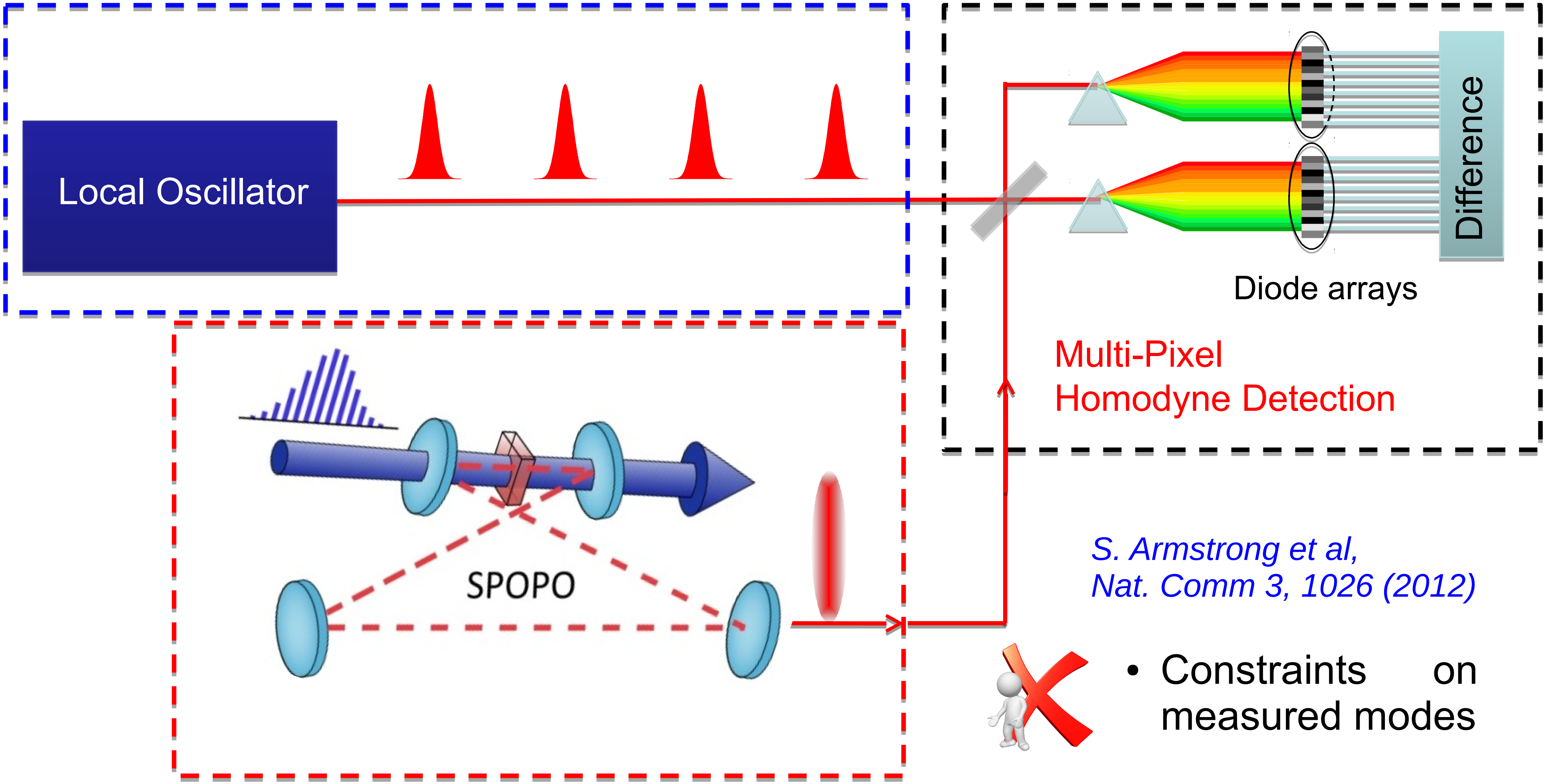
Diagonalization

- Independent Squeezers
- Transition matrix to measured modes



$$\Gamma_{pp,jk} = \frac{1}{2}\langle\{\hat{p}_j, \hat{p}_k\}\rangle$$

Multi-Pixel Homodyne Detection



S. Armstrong et al, Nat. Comm 3, 1026 (2012)

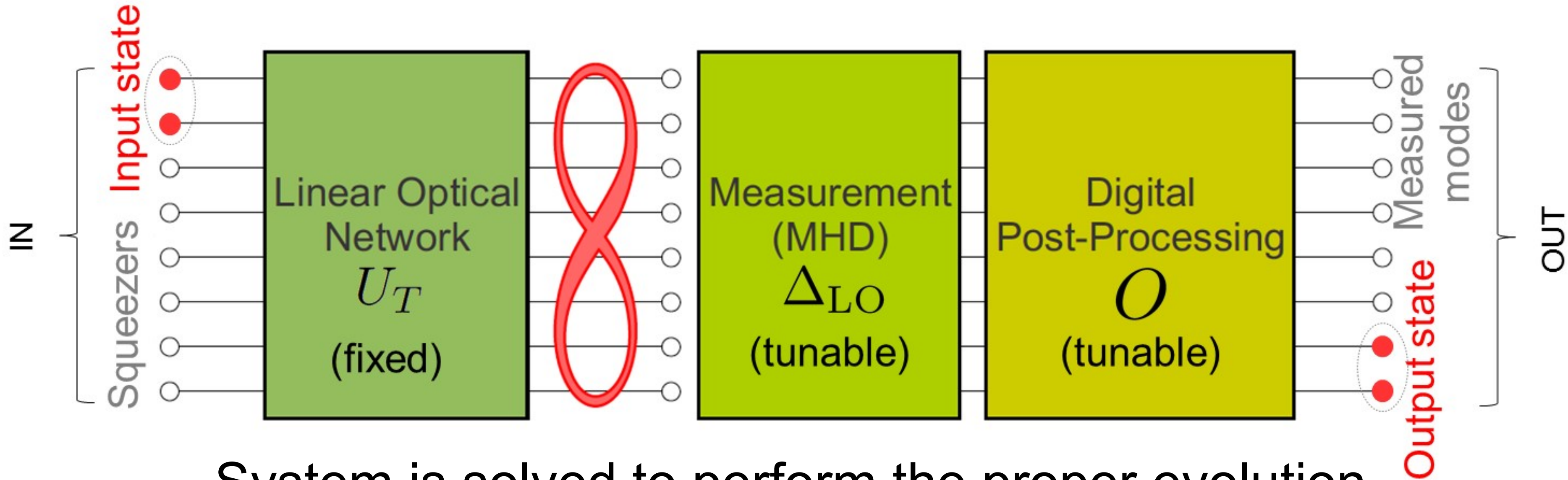
- Constraints on measured modes

- Modes can be separated easily
- Measurement of one mode does not destroy the rest of the system

1)Can we use the state anyway ?

2)Can we engineer correlations given such constraints ?

Direct Measurement-Based QC



System is solved to perform the proper evolution and eliminate "anti-squeezed" quadratures

$$\begin{pmatrix} \vec{x}^{\,\mathrm{out}} \\ \vec{p}^{\,\mathrm{out}} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \vec{x}^{\,\mathrm{in}} \\ \vec{p}^{\,\mathrm{in}} \end{pmatrix} + \begin{pmatrix} \vec{\delta}_x \\ \vec{\delta}_p \end{pmatrix} + \begin{pmatrix} \vec{\eta}_x \\ \vec{\eta}_p \end{pmatrix}$$

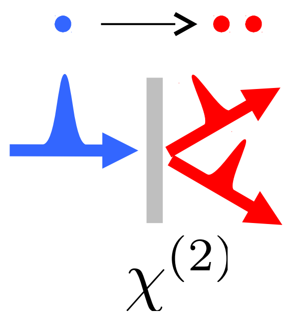
Excess noise due to finite squeezing

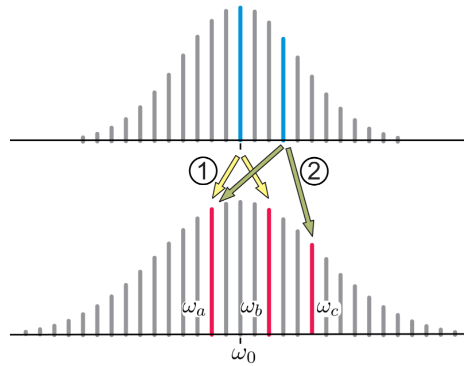
Correction factors from post processing

G. Ferrini et al, PRA 94, 062332 (2016)

- Perform Gaussian gates
- Works for non-Gaussian inputs

Pump Shaping: Experimental Setup

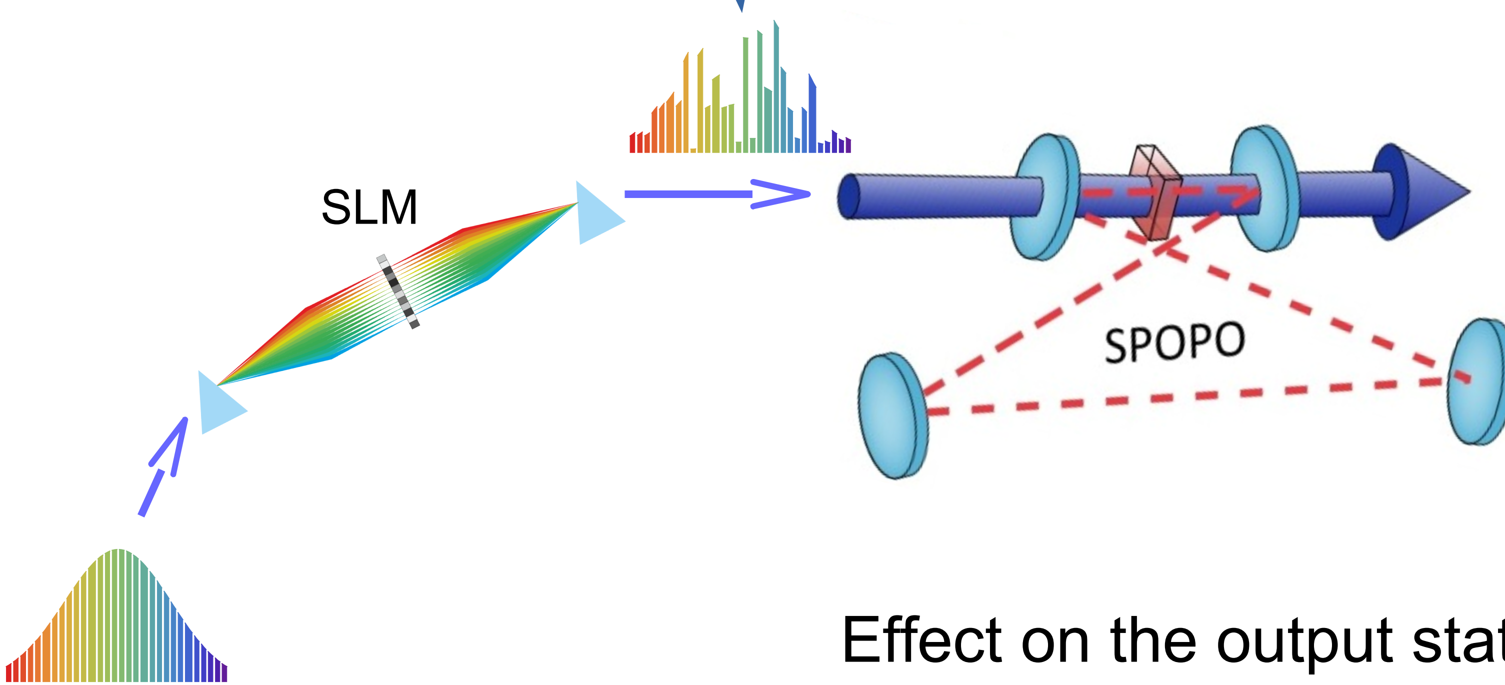




Interaction Hamiltonian

$$H = i \sum_{m,q} \mathcal{L}_{m,q} \hat{a}^\dagger_{\omega_m} \hat{a}^\dagger_{\omega_q} + \text{h.c.}$$

$$\mathcal{L}_{m,q} = \underbrace{\text{sinc}\left(\Delta k_{m,q} \frac{l}{2}\right)}_{\text{Crystal}} \times \underbrace{\alpha\left(\omega_m + \omega_q\right)}_{\text{Pump}}$$



Effect on the output state ?

Tweaking the Squeezing

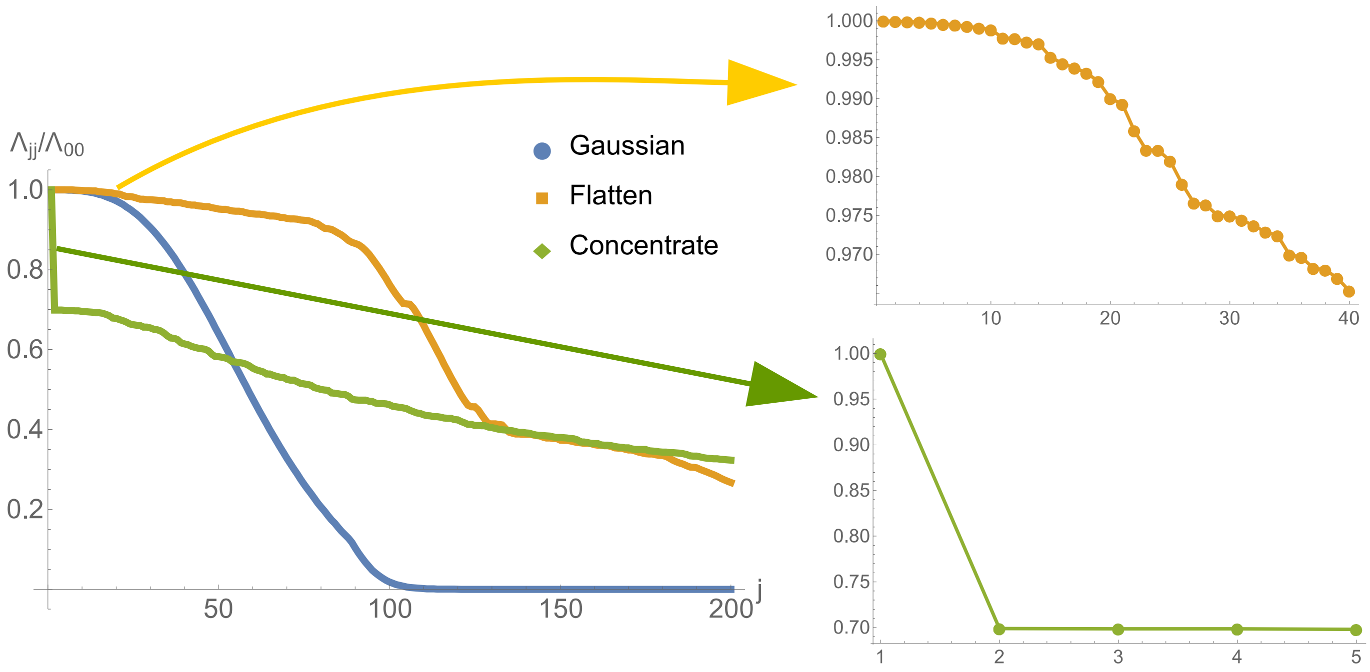
Complex relation between pump and squeezing/supermodes :

Use **numerical optimization**

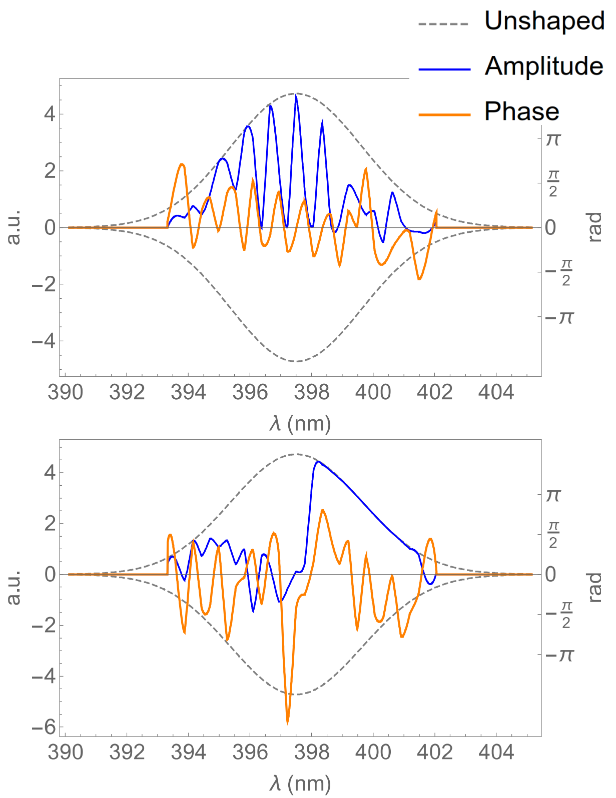
F. Arzani et al,
In preparation...I should be writing.

To flatten the squeezing spectrum : $f_{\mathrm{Fl}}\left(\vec{\theta}\right)=\sum_{j=0}^{100}\Lambda_{jj}\left(\vec{\theta}\right)/\Lambda_{00}\left(\vec{\theta}\right)$

To concentrate the squeezing in one mode : $f_{\mathrm{Conc}}\left(\vec{\theta}\right)=\Lambda_{00}\left(\vec{\theta}\right)/\Lambda_{11}\left(\vec{\theta}\right)$



Optimal Pump

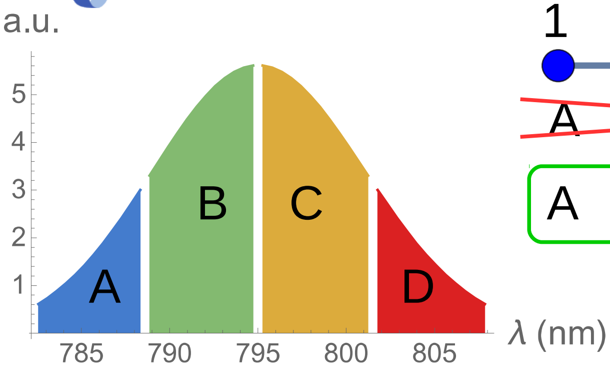


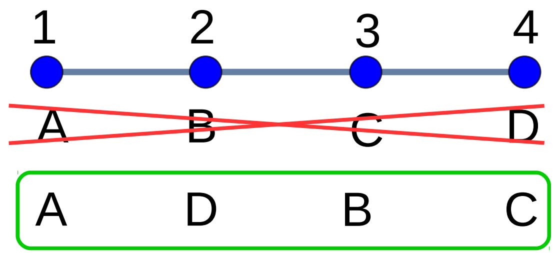
Adjust the squeezing spectrum : Quantum simulation of oscillator networks

With V. Parigi

J. Nokkala et al,
Sci. Rep. 6, 26861 (2016)

Optimizing CV Cluster States - QC





$$\hat{\delta}_1 = \hat{p}_A - \hat{q}_D$$

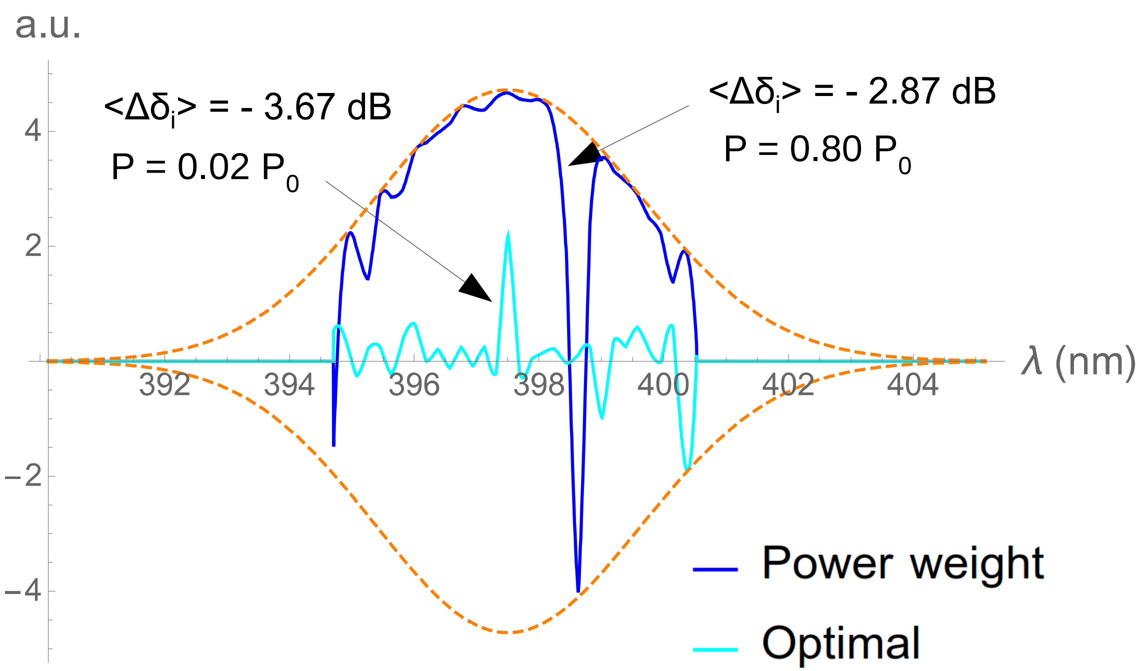
Mean nullifiers' squeezing :

$\langle \Delta\delta_i \rangle$ = - 0.18 dB

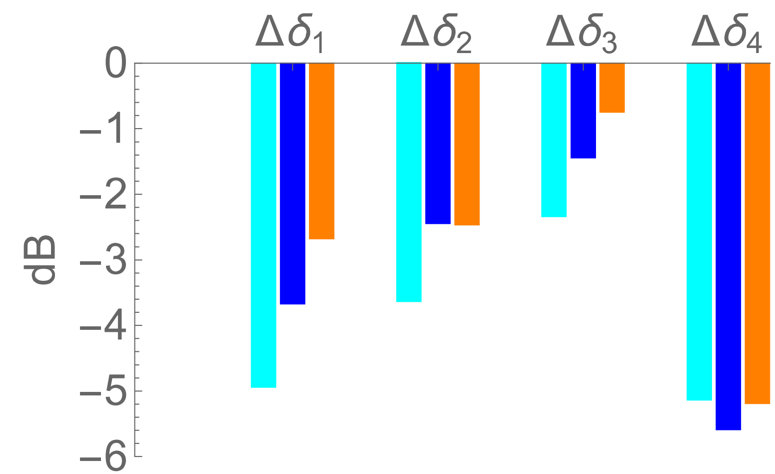
$\langle \Delta\delta_i \rangle$ = - 2.31 dB

Fully inseparable (PPT)

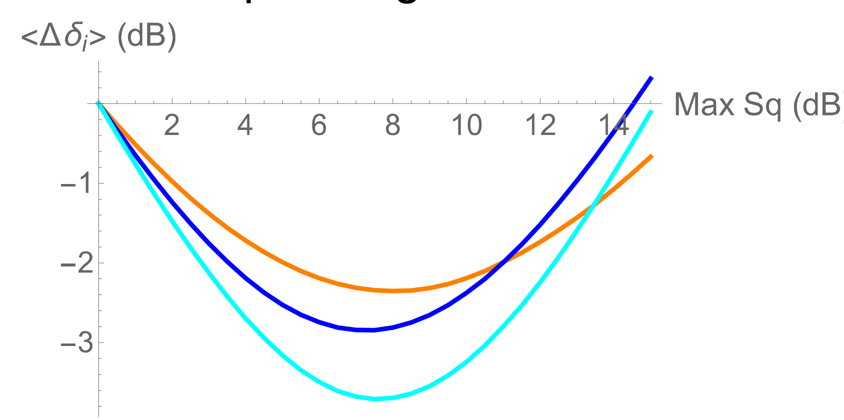
Pump optimizing average nullifiers' noise
(fixed maximum squeezing = 7 dB)



Nullifiers' squeezing

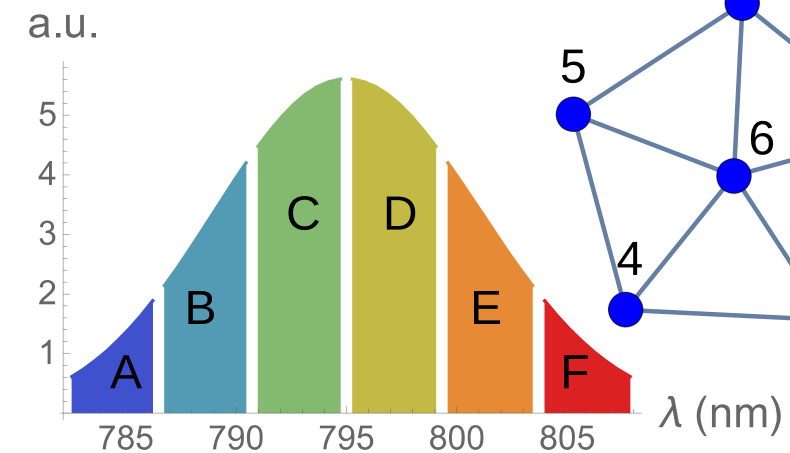


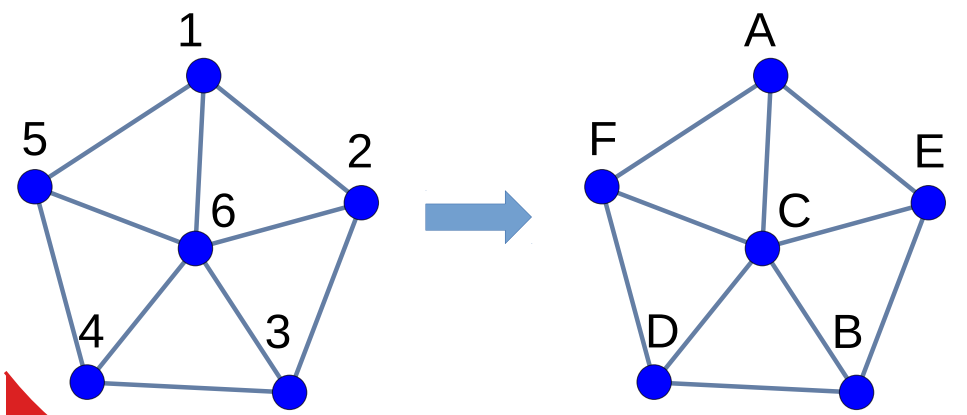
More squeezing ≠ Better nullifiers



Non trivial spectral phase as well...

Optimizing CV Cluster States - SS



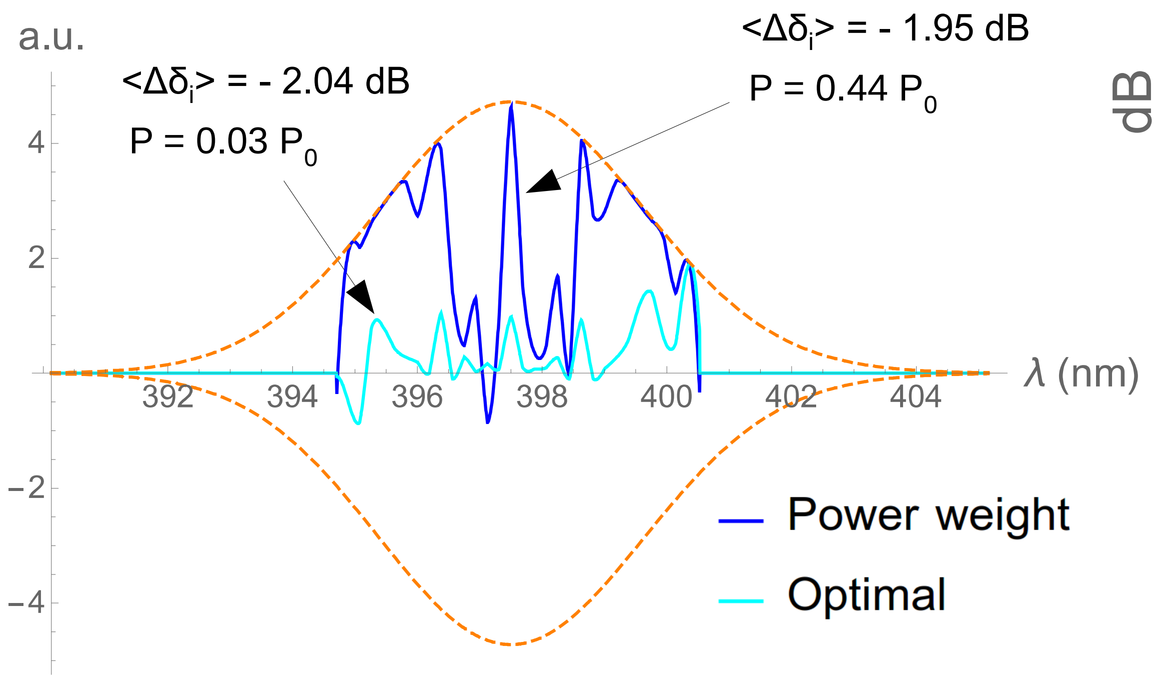


Mean nullifiers' squeezing :

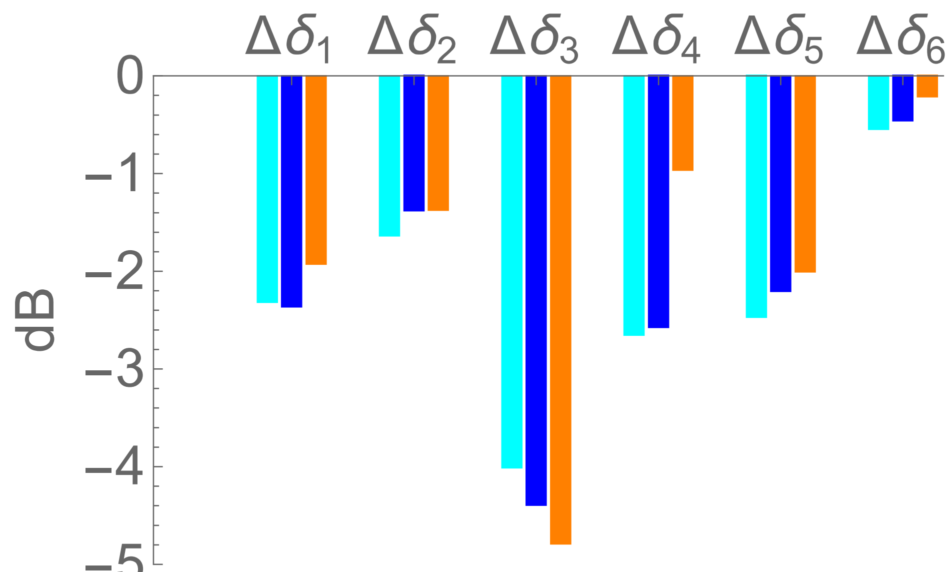
$\langle \Delta\delta_i \rangle$ = - 1.57 dB

Fully inseparable (PPT)

Pump optimizing average nullifiers' noise
(fixed maximum squeezing = 7 dB)



Nullifiers' squeezing



	Purity
Gaussian	0.73
Optimal PW	0.84
Optimal	0.91

Non-Gaussian Gates

Universal set for any evolution with
polynomial hamiltonians in the quadratures

S. Lloyd and S. L. Braunstein
PRL 82, 1784 (1999)

$$\left\{ e^{i\hat{q}s}, e^{i\hat{q}^2 s}, e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, e^{i\hat{q}_1\otimes\hat{q}_2}, e^{i\hat{q}^3 s} \right\}$$

Single-mode, Gaussian

Two-mode C_Z

Non-Gaussian

Universality:

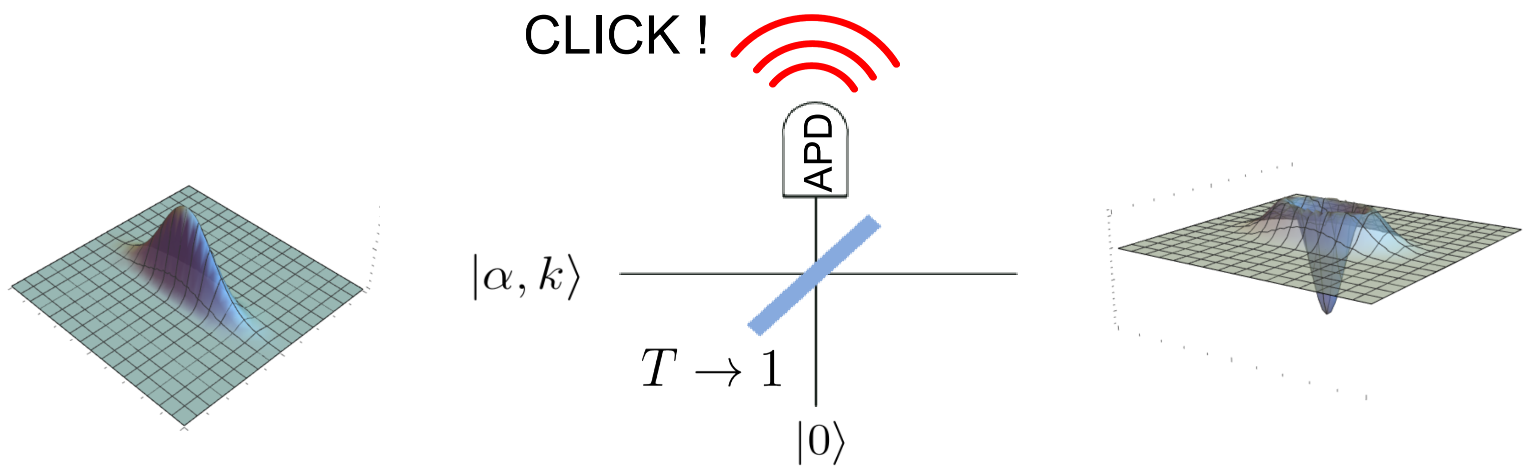
$$e^{i\hat{A}\delta t}e^{i\hat{B}\delta t}e^{-i\hat{A}\delta t}e^{-i\hat{B}\delta t} = e^{\left(\hat{A}\hat{B}-\hat{B}\hat{A}\right)\delta t^2} + O(\delta t^3)$$

No quantum advantage without non-Gaussianity !

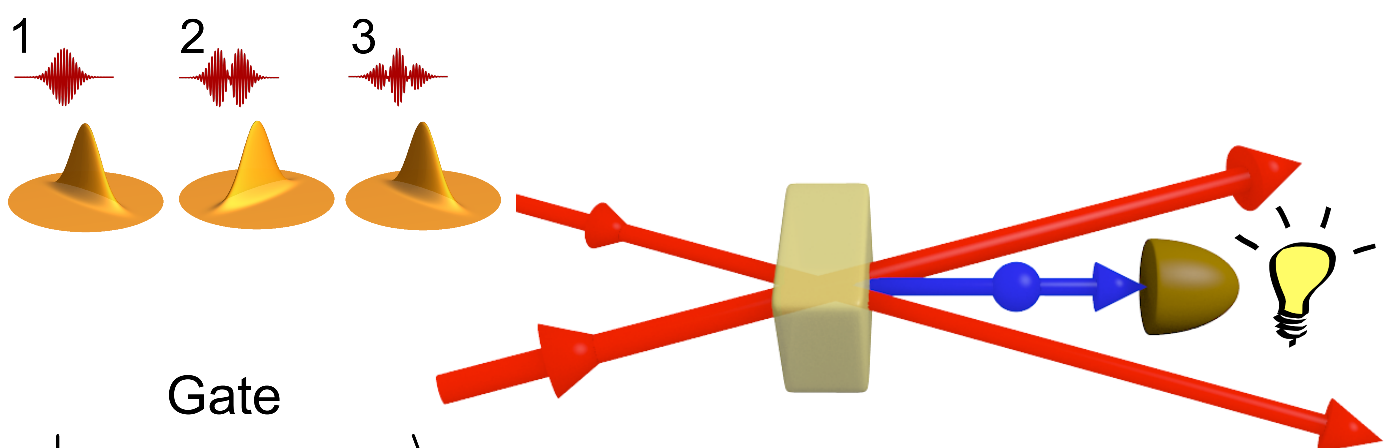
A. Mari and J. Eisert
PRL 109, 230503 (2012)

Actually, it's negativity of the WF...

Introducing Negativity: Photon Subtraction



Multimode: sum-frequency generation

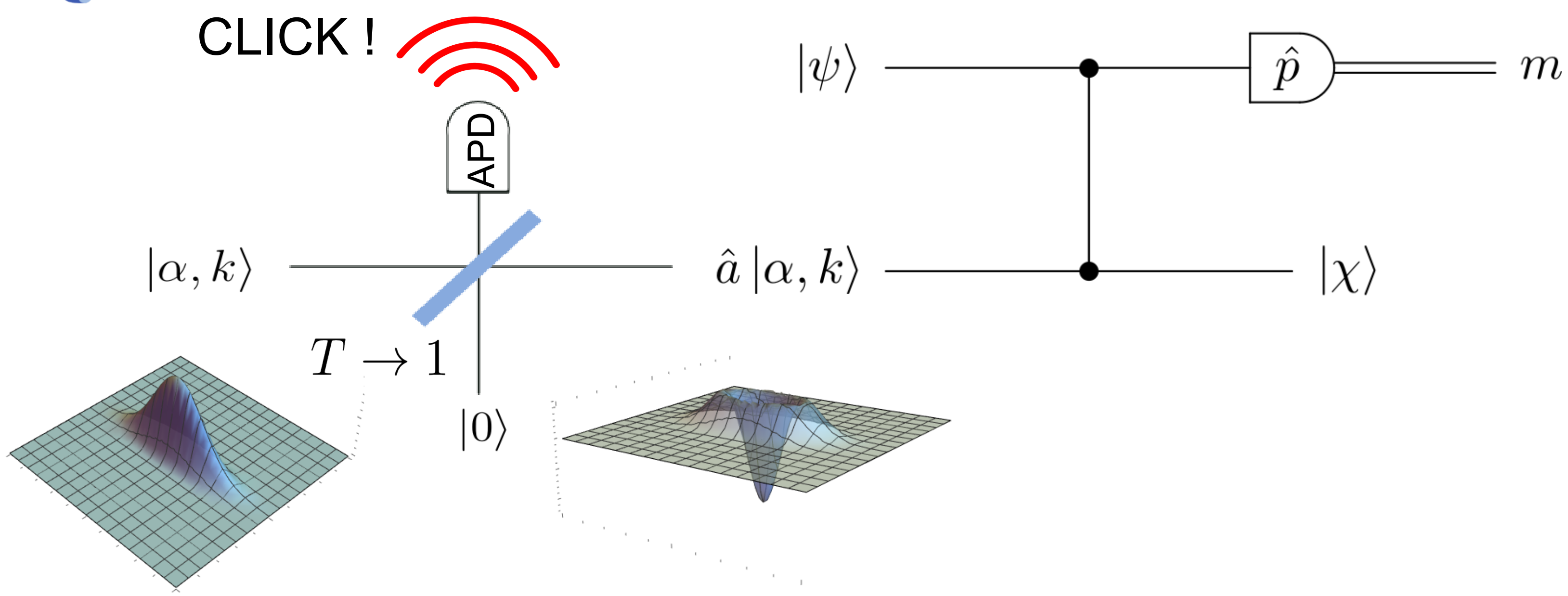


$$|\alpha_1, \alpha_2, \ldots\rangle$$

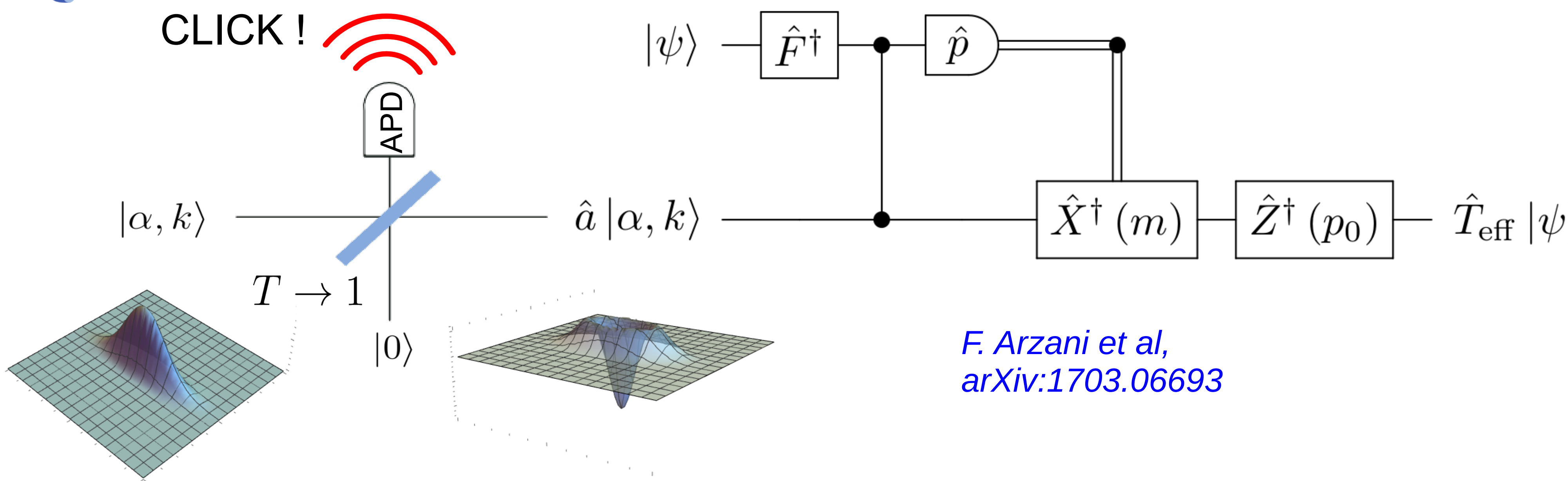
Mode selectivity!

$$\propto (\alpha_1 \hat{a}_1 + \alpha_2 \hat{a}_2 + \ldots)|\psi_s\rangle$$

Introducing Negativity: Photon subtraction



Polynomials through Photon Subtracted Ancillae



F. Arzani et al,
arXiv:1703.06693

$$\hat{T}_{\mathrm{eff}} = \mathcal{N} \exp\left\{-\frac{(\hat{q} - q_0 + m)^2}{k^2}\right\} \left(\hat{q} - \lambda(\alpha, k, m)\right)$$

Normalization — ~ attenuation — Monomial in *q*

$$\lambda(\alpha, k, m) = -\left(\frac{2}{k^2 - 2}\right) q_0 - i\left(\frac{k^2}{k^2 - 2}\right) p_0 - m$$

<u>Depends on</u>

- Experimental parameters
- Measurement (!!!)

LKB

Approximating Unitaries

- Repeated application: polynomial in the quadratures

P. Marek et al
PRA 84(5), 053802 (2011)

K. Marshall et al
PRA 91, 032321 (2015)

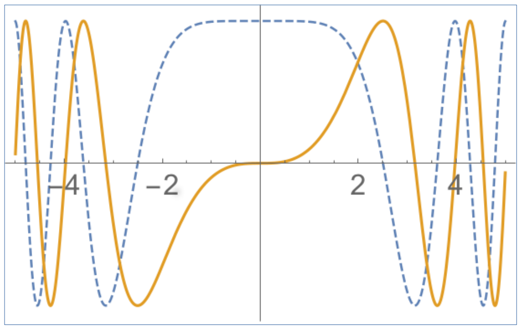
es.

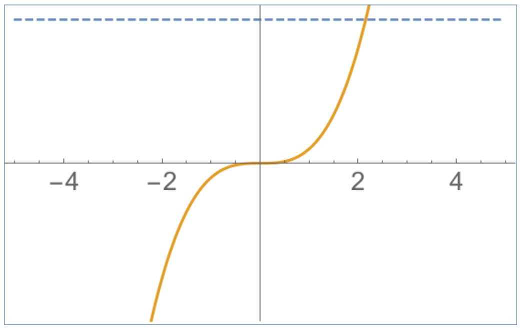
$$e^{i\nu\hat{q}^3} \approx \mathbb{I} + i\nu\hat{q}^3 = (\hat{q} - \lambda_1)(\hat{q} - \lambda_2)(\hat{q} - \lambda_3)$$

- Success probability exponentially drops with the degree
- Deterministic implementation: prepare a resource state offline

Benchmarking: Fidelity of the Bare Polynomial

$e^{0.1ix^3}$ $\qquad$ $1+0.1ix^3$

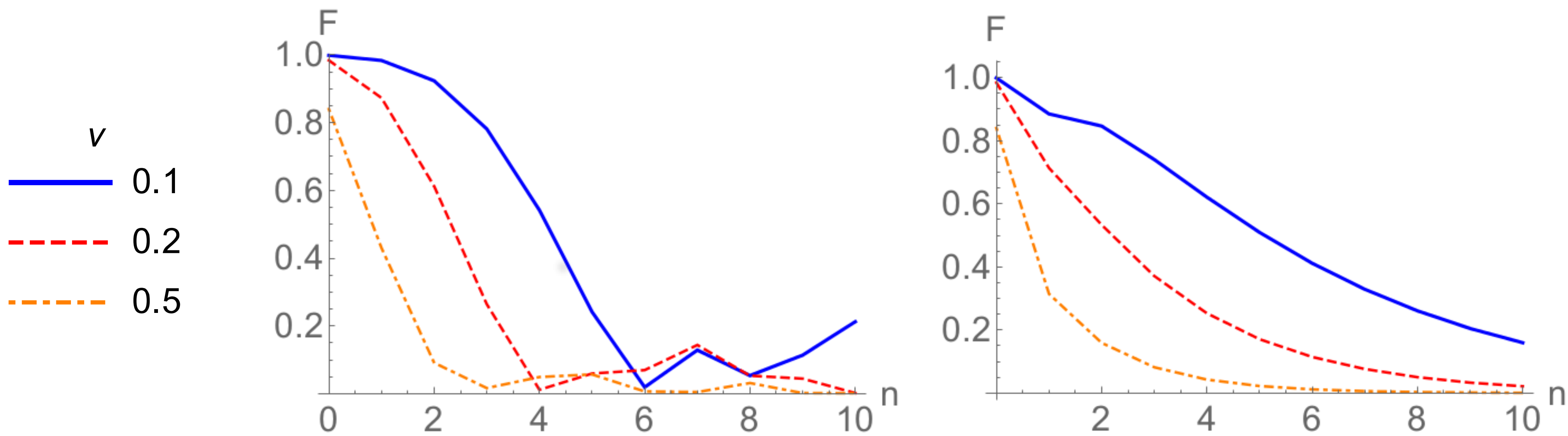




$$\mathcal{F} = \left| \langle\psi|\, \hat{U}^\dagger \hat{\mathcal{T}} |\psi\rangle \right|$$

$$\hat{U} = e^{i\nu\hat{q}^3} \qquad \hat{\mathcal{T}} = \mathcal{N}_\psi \left(\mathbb{I} + i\nu\hat{q}^3\right)$$

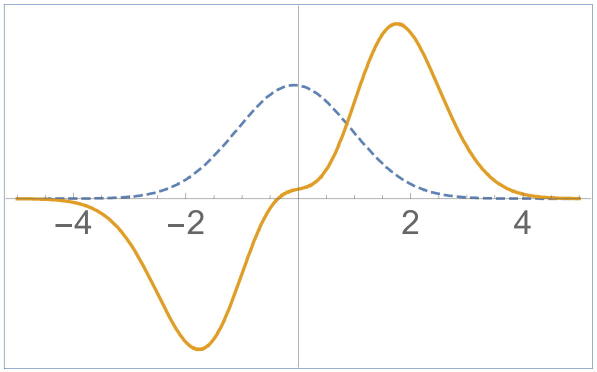
Non unitary: ψ-dependent normalization



Input: Fock states Coherent states

Benchmarking: Fidelity

$$\mathcal{T}_{\rm eff}\left(x,\vec{m}\right)=\prod_{i=1}^{3}\left[\mathcal{G}\left(x,\alpha_i,k,m_i\right)\left(x-\lambda\left(\alpha_i,k,m_i\right)\right)\right]$$



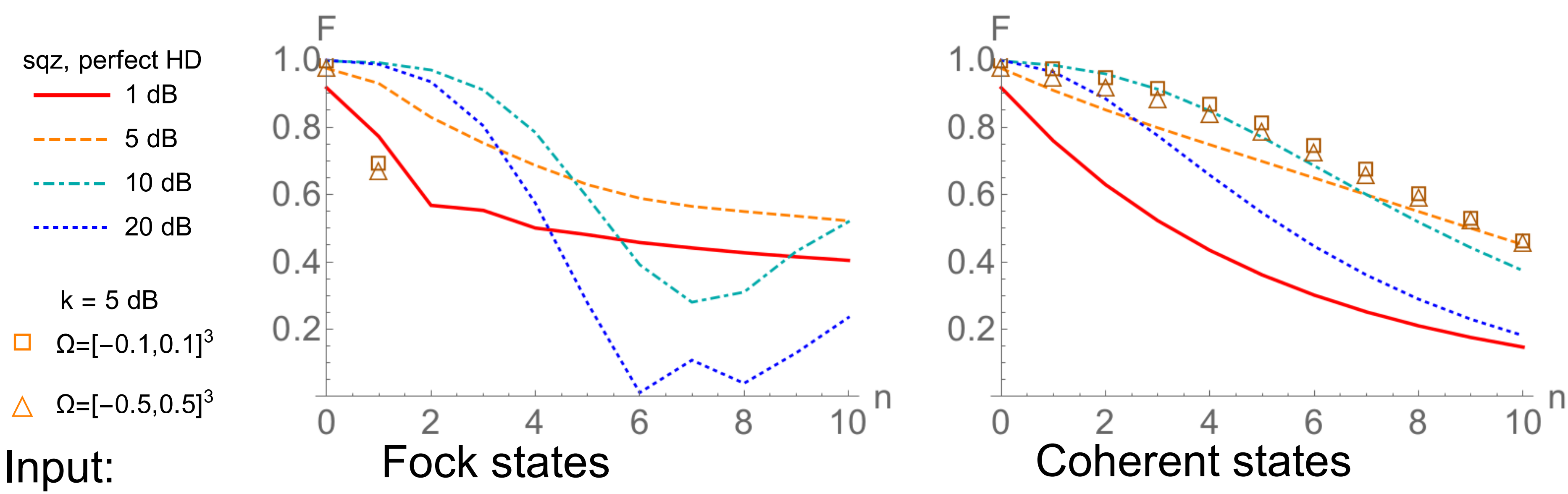
- Finite squeezing: envelope
- Imperfect measurement: deformation
- Finite **success probability**: acceptance region Ω

≈ 10^{-9} - 10^{-12}

Average output state

$$\rho_\Omega=\int_\Omega \mathrm{d}^n m\frac{p\left(\vec{m}\right)}{p_\Omega}\hat{\mathcal{T}}_{\rm eff}\left(\vec{m}\right)\left|\psi\right\rangle\left\langle\psi\right|\hat{\mathcal{T}}_{\rm eff}^\dagger\left(\vec{m}\right)$$

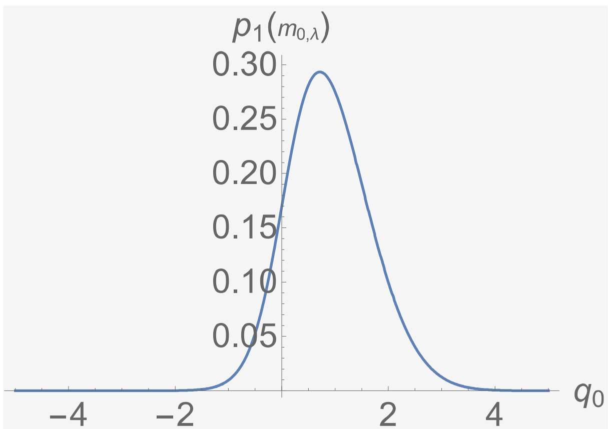
$$\mathcal{F}=\sqrt{\left\langle\psi\right|\hat{U}^\dagger\rho\hat{U}\left|\psi\right\rangle}$$



State Preparation

- Know the input: optimize displacements to increase probability

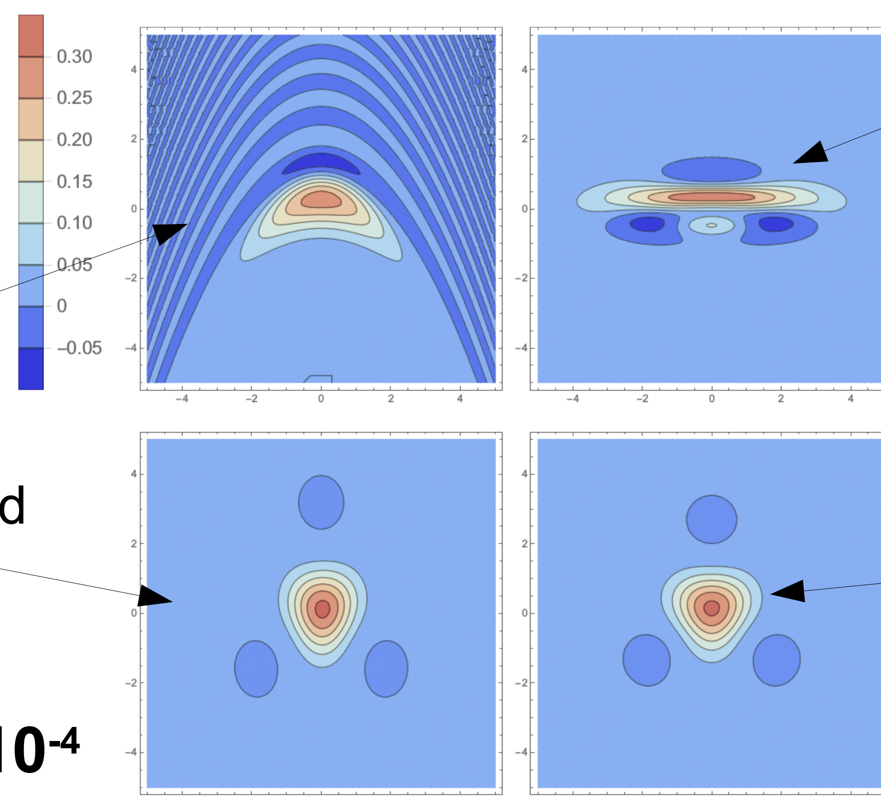
$$\left(\hat{q}-\lambda\left(\alpha,k,m\right)\right) \rightsquigarrow \lambda\left(\alpha,k,m\right)=-\left(\frac{2}{k^2-2}\right)q_0-i\left(\frac{k^2}{k^2-2}\right)p_0-m$$



Cubic phase state

$$|\gamma\left(\nu\right)\rangle=\hat{\gamma}\left(\nu\right)|0\rangle_p\rightarrow\hat{\gamma}_{\mathrm{appr}}\left(\nu\right)|k\rangle_p$$

Wigner function (k = 5 dB, v = 0.1)



Bare polynomial

$$\hat{U}=e^{0.1i\hat{q}^3}$$

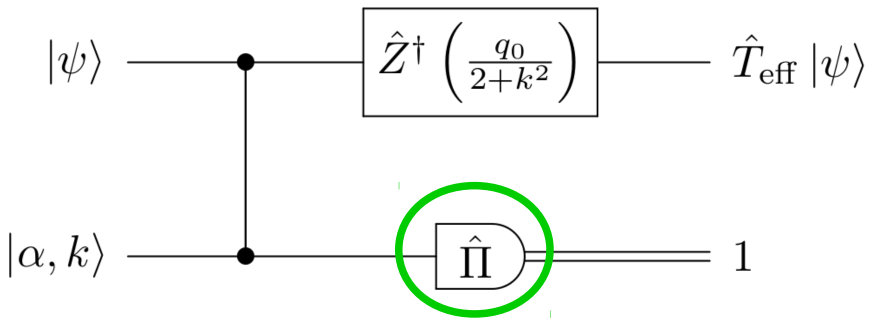
F = 0.9

Photon-subtracted ancilla

Next slide

Success probability ~ 10^{-2} - 10^{-4}

Alternative Scheme: Single-Photon Counter



Projects on **single photon**

Still post-selection but no binning of HD outcomes

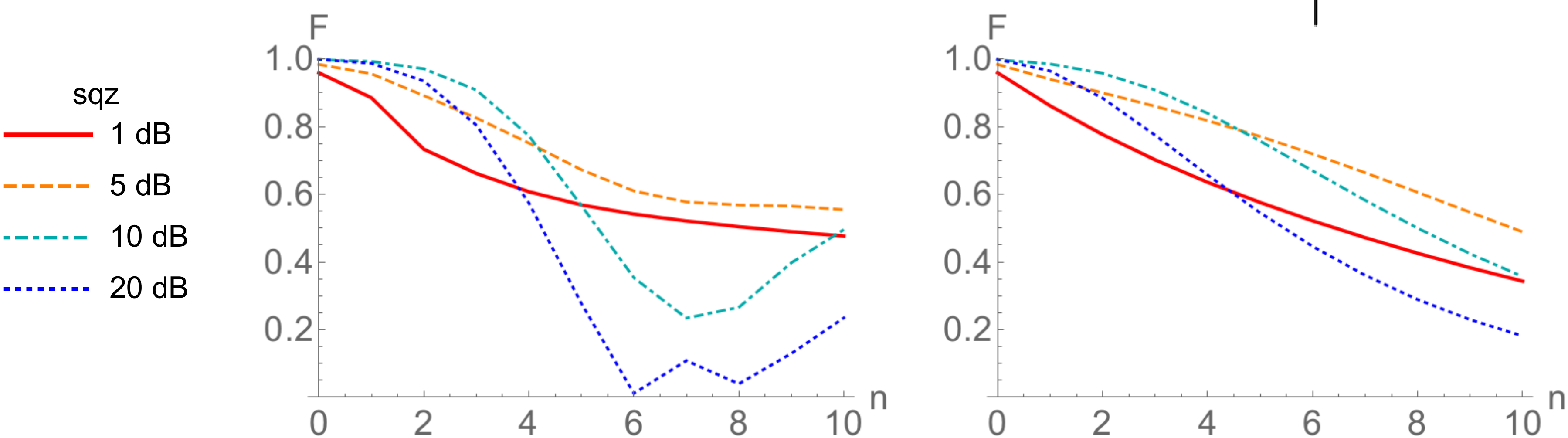
~ Conjugate process of

K. Park et al PRA 90, 013804 (2014)

$$\hat{T}_{\text{eff}} = \tilde{\mathcal{N}} \exp\left\{-\left(\frac{k^2}{4+2k^2}\right)(\hat{q}+p_0)^2\right\}\left(\hat{q}-\lambda(\alpha,k)\right)$$

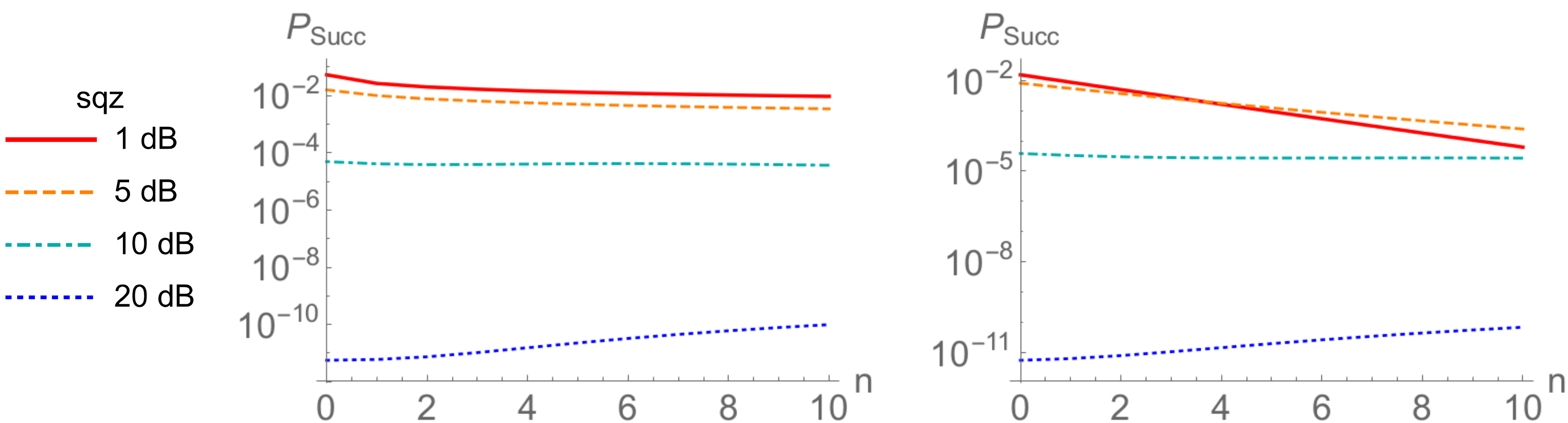
$$\lambda(\alpha,k) = \frac{2i}{k^2}q_0 - p_0$$

$$\mathcal{F} = \left|\langle\psi|\,\hat{U}^{\dagger}\hat{\mathcal{T}}|\psi\rangle\right|$$



Input: Fock states Coherent states

Success Probability



Input: Fock states Coherent states

- Probability of detecting exactly one photon in each step
- Degrades for high squeezing: many photons from squeezing, displacement

Summary

- Modes, many many modes, infinite dimensions for QIP
- Multi-pixel homodyne for (lazy) MBQC
- Shaping the pump for better cluster states, a better world
- Non-Gaussianity subtracting photons, counting one photon at a time

Thank you !