Polynomial approximation of non-Gaussian unitaries by counting one photon at a time

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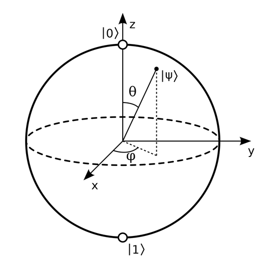
Abstract

In quantum computation with **continuous-variable** systems, quantum advantage can only be achieved if some **non-Gaussian** resource is available. Yet, non-Gaussian unitary evolutions and measurements suited for computation are challenging to realize in the laboratory. We propose and analyze two methods to apply a **polynomial approximation** of any unitary operator diagonal in the amplitude quadrature representation, including non-Gaussian operators, to an unknown input state. Our protocols use as a primary non-Gaussian resource a single-photon counter. We use the **fidelity of the transformation** with the target one on **Fock and coherent states** to assess the quality of the approximate gate.

F. Arzani, N. Treps, G. Ferrini, Phys. Rev. A **95**, 052352 (2017)

Quantum Information and computing with CV

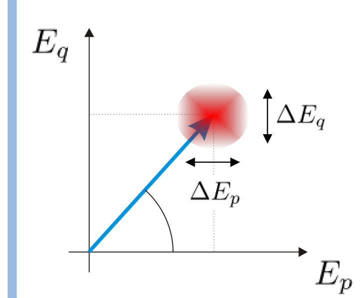
DV : information encoded in qubits



Optical ex:
Polarization of single photon

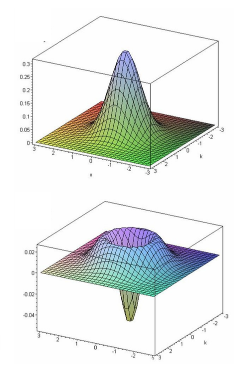
CV : information encoded in observables with continuous spectrum, e.g. : $\hat{q},\ \hat{p}$

Optical ex:
Field quadratures



$\hat{E}_Q \propto \hat{a} + \hat{a}^\dagger$

$\hat{E}_P \propto i(\hat{a}^\dagger - \hat{a})$



Wigner function

$$W(q,p) = \frac{1}{2\pi}\int \mathrm{d}x e^{ipx}\left\langle q - \frac{x}{2}\right|_q \hat{\rho}\left|q + \frac{x}{2}\right\rangle_q$$

For **pure states**: Gaussian iff Non-negative

Discrete encoding $\vec{x} = 1001111010101001\ldots$

bits $\vec{x} \mapsto b(\vec{x})$ For any boolean function

qubits $|\vec{x}\rangle|\phi\rangle \mapsto U_b|\vec{x}\rangle|\phi\rangle = |\vec{x}\rangle|b(\vec{x})\rangle$

Continuous Variables [2]

qumodes $|\psi\rangle \in \left(L^2(\mathbb{R},\mathbb{C})\right)^{\otimes n} \mapsto e^{-itH(\hat{a},\hat{a}^\dagger)}|\psi\rangle$

General polynomial

Computation with arbitrary encoding

Universal set:

$$\left\{e^{i\hat{q}s}, e^{i\hat{q}^2 s}, e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, e^{i\hat{q}_1\otimes\hat{q}_2}, e^{i\hat{q}^3 s}\right\}$$

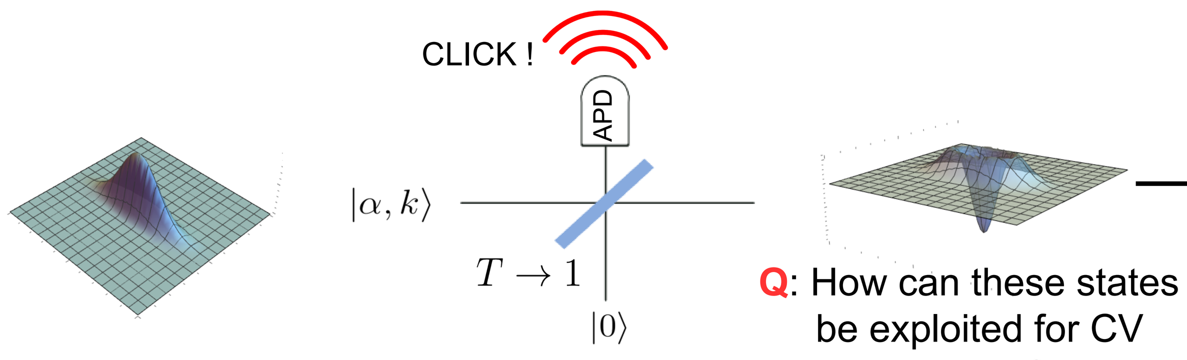
Single-mode, Gaussian — Two-modes C_Z — Non-Gaussian

No quantum advantage without non-Gaussianity !

Actually, it's negativity of the WF [3]...

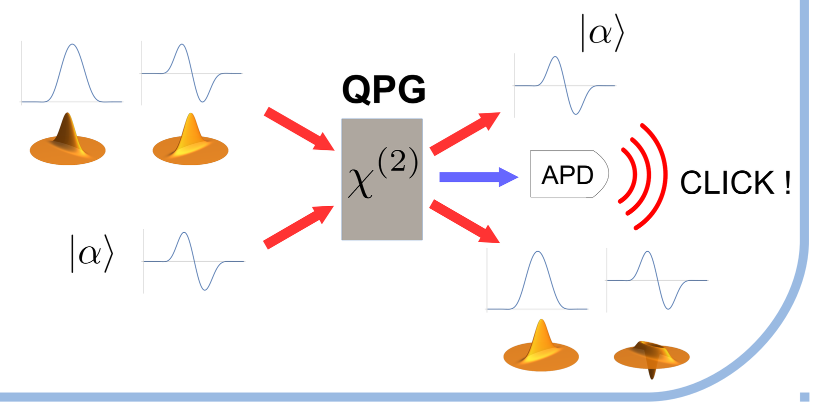
Negativity from photon subtraction

Negative Wigner functions can be obtained in the lab subtracting single photons from vacuum squeezed states

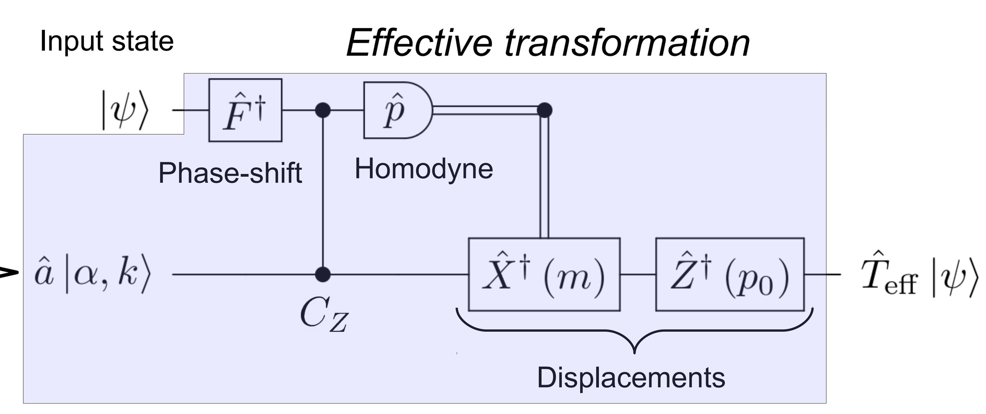


Q: How can these states be exploited for CV computation?

Why we bother:
Experiments in LKB for mode-selective photon subtraction from a multi-mode squeezed state



Polynomial operations



$$\hat{T}_{\mathrm{eff}} = \mathcal{N}\exp\left\{-\frac{(\hat{q}-q_0+m)^2}{k^2}\right\}\left(\hat{q} - \lambda(\alpha,k,m)\right)$$

Normalization — Gaussian envelope — Monomial in *q*

A: An approximate monomial transformation can be applied to the input state

Repeating the box allows to build **polynomial functions** of the position operator [4], modulo a Gaussian factor.

Each **monomial** depends on the **experimentally tunable** displacement α and squeezing and on the random **measurement ouctome *m***.

Tuning α and *k* and **post-selecting** on the right *m*, with three applications of the box one could approximate *e.g.*

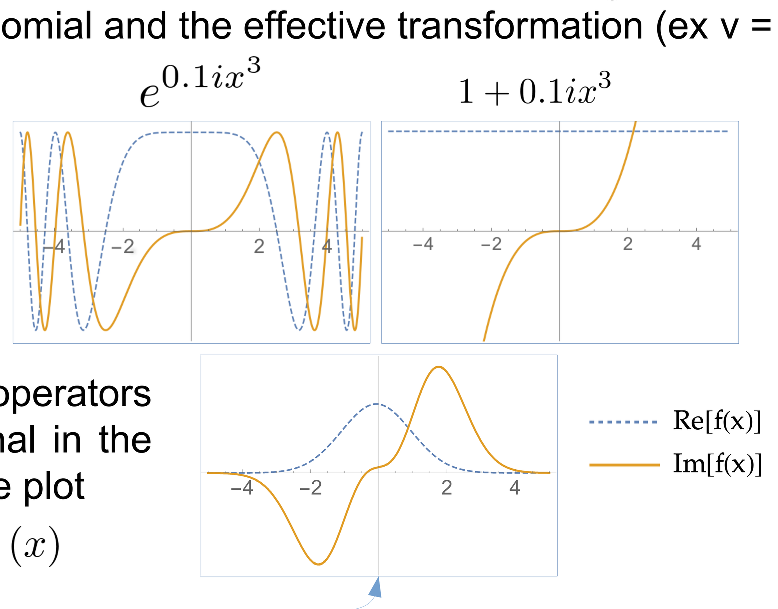
$$e^{i\nu\hat{q}^3} \approx \mathbb{I} + i\nu\hat{q}^3 = (\hat{q}-\lambda_1)(\hat{q}-\lambda_2)(\hat{q}-\lambda_3)$$

Homodyne projects on a continuous space, so for post-selection one has to introduce an **acceptance region** Ω and consider the **average state**

$$\rho_\Omega = \int_\Omega \mathrm{d}^n m \frac{p(\vec{m})}{p_\Omega}\hat{\mathcal{T}}_{\mathrm{eff}}(\vec{m})\,|\psi\rangle\langle\psi|\,\hat{\mathcal{T}}^\dagger_{\mathrm{eff}}(\vec{m})$$

Benchmarking the approximation

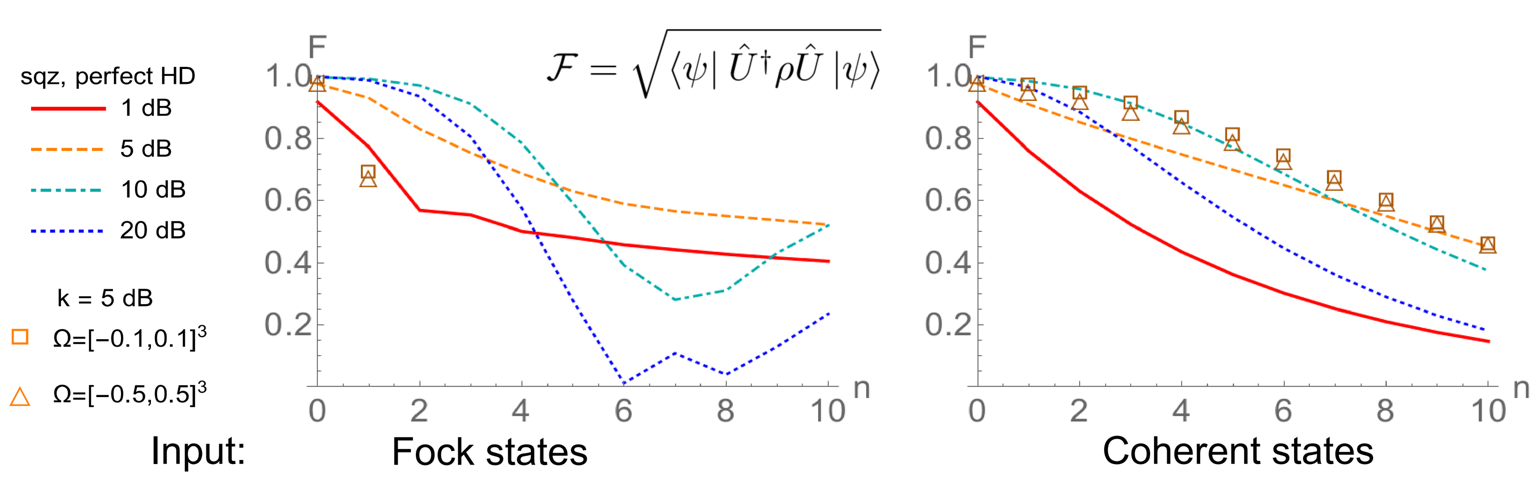
Position representation of the ideal gate, the bare polynomial and the effective transformation (ex v = 0.1)



Since all operators are diagonal in the position we plot $f(\hat{q}) \to f(x)$

$$\mathcal{T}_{\mathrm{eff}}(x,\vec{m}) \propto \prod_{i=1}^{3}\left[\mathcal{G}(x,\alpha_i,k,m_i)(x-\lambda(\alpha_i,k,m_i))\right]$$

Fidelity between the average state obtained with the effective transformation approximating $\hat{U} = e^{i\nu\hat{q}^3}$ to third order



Lines: single homodyne outcome (m_i takes the correct value)
Points: finite acceptance regions (m_i is averaged over Ω)

State preparation

The success probability is low (≈ 10^{-9} – 10^{-12}):

Instead of applying the gate to unknown inputs, use the box on known states to **produce resource states**.

Know the input: **otimize** the experimental parameters and increase the **success probability**:

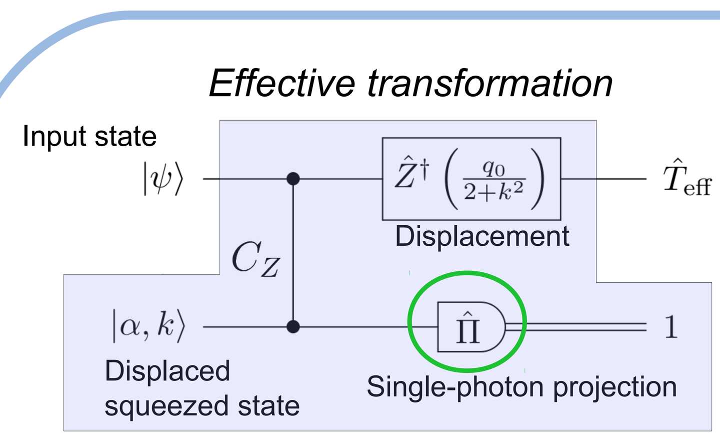
$$\lambda(\alpha,k,m) = -\left(\frac{2}{k^2-2}\right)q_0 - i\left(\frac{k^2}{k^2-2}\right)p_0 - m$$

For $|\gamma(\nu)\rangle = \hat{\gamma}(\nu)|0\rangle_p \to \hat{\gamma}_{\mathrm{appr}}(\nu)|k_{in}\rangle_p$

Fidelity = 0.9 with $\hat{\gamma}(\nu)|k_{in}\rangle_p = e^{i\nu\hat{q}^3}|k_{in}\rangle_p$ for k = 5 dB, k_{in} = 5 dB, v = 0.1

Success probability ~ 10^{-4}

Alternative scheme



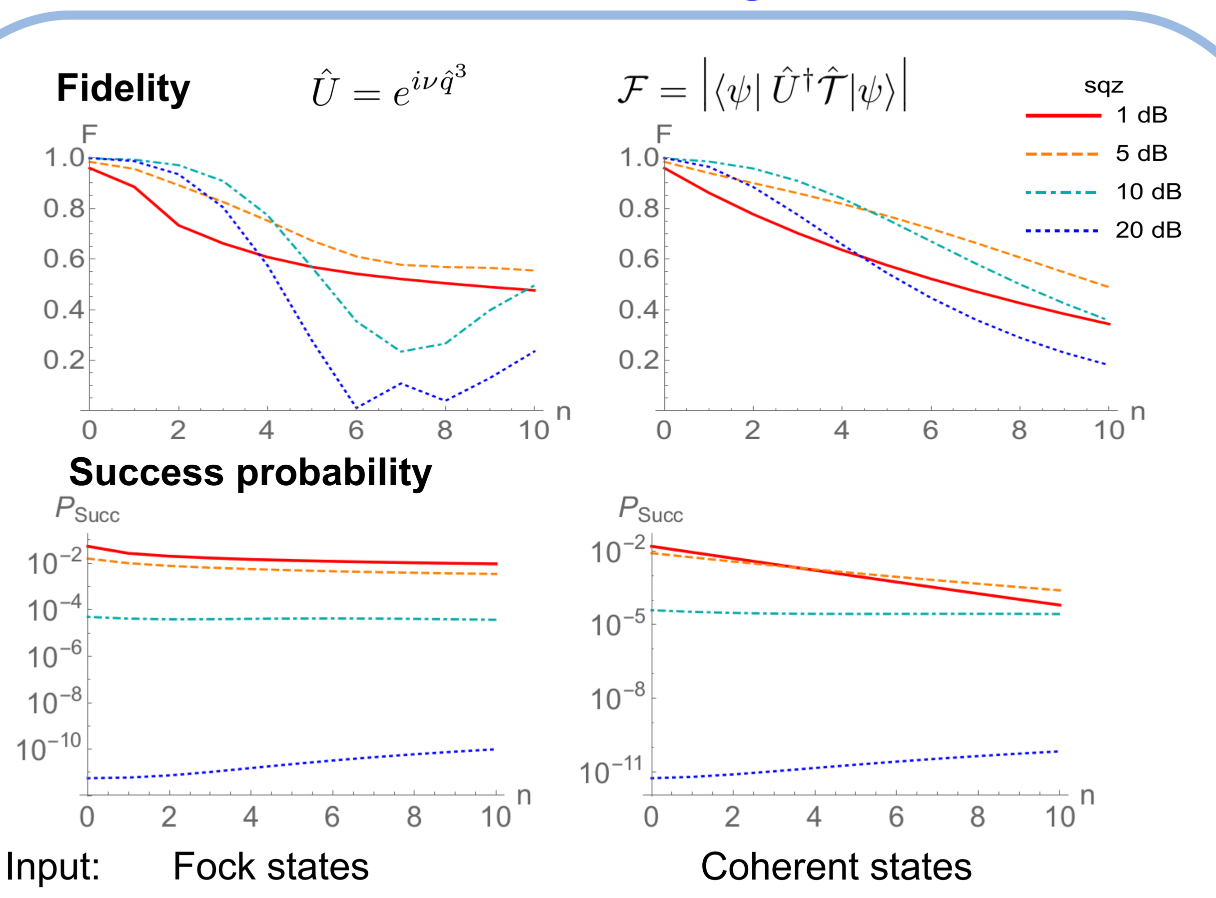
$$\hat{T}_{\mathrm{eff}} = \tilde{\mathcal{N}}\exp\left\{-\left(\frac{k^2}{4+2k^2}\right)(\hat{q}+p_0)^2\right\}\left(\hat{q} - \lambda(\alpha,k)\right)$$

Still requires post-selection but **no binning** of continuous parameters

$$\lambda(\alpha,k) = \frac{2i}{k^2}q_0 - p_0$$

Fully specified by tunable parameters

Benchmarking



Conclusions

We presented two probabilistic protocols for engineering arbitrary evolutions by means of a polynomial approximation.
Both may be achieved with existing technology.
We find low success probabilities for the first protocol, but these can be increased optimizing the protocol for state preparation.
The second protocol has slightly higher success probabilities and could directly be incorporated in measurement-based algorithms with CV cluster states.
Both schemes are also conceptually interesting as they can be used for sub-universal setups based on post-selection, such as CV instantaneous quantum computing.

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References:
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[4]P. Marek, R. Filip, A. Furusawa, *Phys. Rev. A* **84** (5), 053802