Quantum computing with squeezing, homodyne and clicks

Laboratoire Kastler Brossel
Physique quantique et applications

Francesco Arzani, Giulia Ferrini, Nicolas Treps

LETTRES SCIENCES ECOLE NORMALE SUPERIEURE

PSL RESEARCH UNIVERSITY

université PARIS DIDEROT PARIS 7

cnrs dépasser les frontières

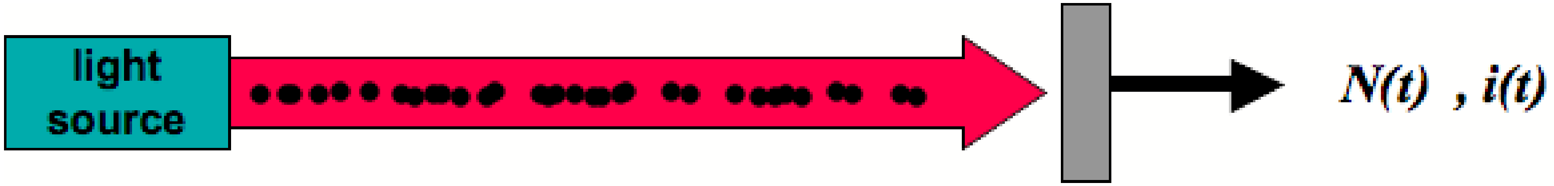
AGENCE NATIONALE DE LA RECHERCHE ANR

Outline

- Continuous variable systems
- Measurement based computing
- Non gaussian transformations

Quantum EM field

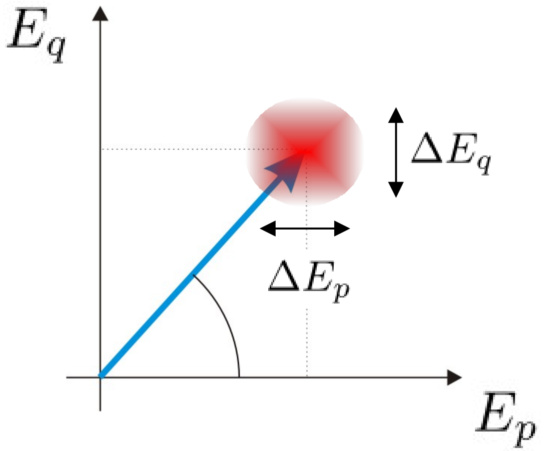
Discrete description : “clicks” of detectors



Quantum EM field

Discrete description : "clicks" of detectors

Continuous description : measure mean values and fluctuations of the field



Quadratures

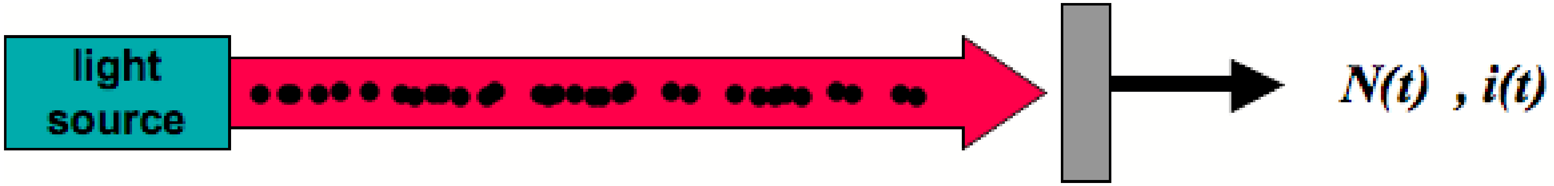
$$\hat{E}_Q \propto \hat{a} + \hat{a}^\dagger$$

$$\hat{E}_P \propto i(\hat{a}^\dagger - \hat{a})$$

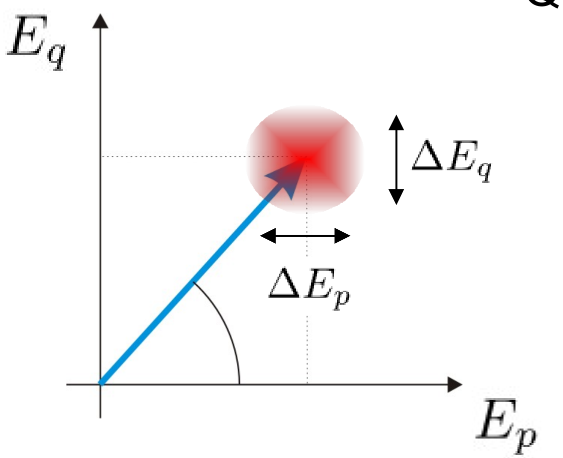
Position and momentum of a harmonic oscillator

Quantum EM field

Discrete description : “clicks” of detectors



Continuous description : measure mean values and fluctuations of the field



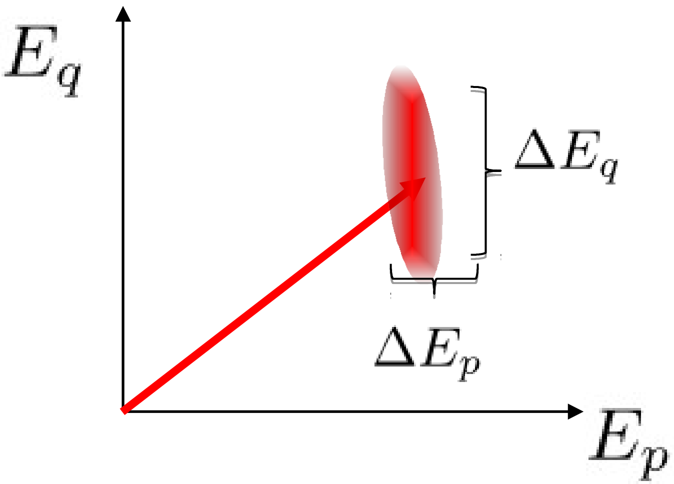
Quadratures

$$\hat{E}_Q \propto \hat{a} + \hat{a}^\dagger$$

$$\hat{E}_P \propto i(\hat{a}^\dagger - \hat{a})$$

Position and momentum of a harmonic oscillator

Squeezed states



Wigner functions

$$W\left(q,p\right)=\frac{1}{2\pi}\int \mathrm{d}x e^{ipx}\left\langle q-\frac{x}{2}\right|_{q}\hat{\rho}\left|q+\frac{x}{2}\right\rangle_{q}$$

Wigner functions

$$W(q,p) = \frac{1}{2\pi}\int \mathrm{d}x e^{ipx} \left\langle q - \frac{x}{2}\right|_q \hat{\rho} \left| q + \frac{x}{2} \right\rangle_q$$

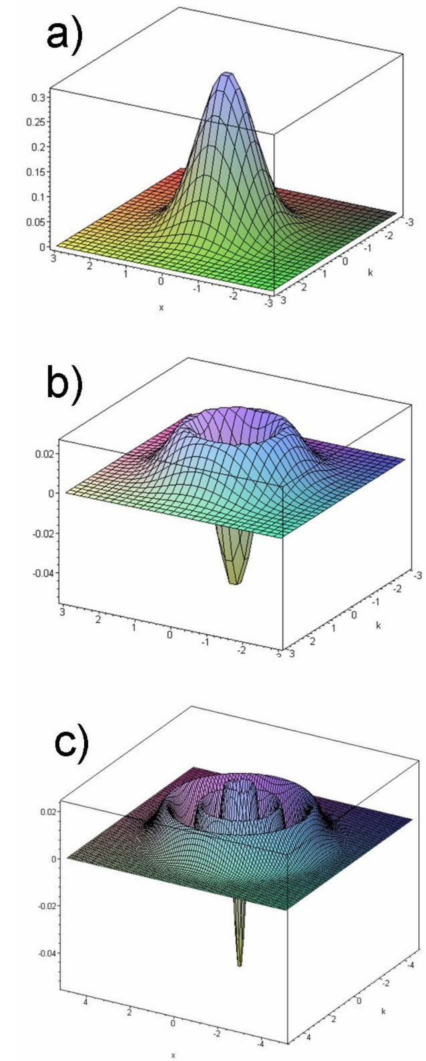
$$\Rightarrow \left\langle \hat{O} \right\rangle = \mathrm{Tr}\left[\hat{\rho}\hat{O}\right] = \int \mathrm{d}q\mathrm{d}p W_\rho(q,p) W_O(q,p)$$

Wigner functions

$$W\left(q,p\right)=\frac{1}{2\pi}\int \mathrm{d}x e^{ipx}\left\langle q-\frac{x}{2}\right|_{q}\hat{\rho}\left|q+\frac{x}{2}\right\rangle_{q}$$

$$\left\langle \hat{O}\right\rangle = \mathrm{Tr}\left[\hat{\rho}\hat{O}\right]=\int \mathrm{d}q\mathrm{d}pW_{\rho}\left(q,p\right)W_{O}\left(q,p\right)$$

- **May be negative** (e.g. for single photons)



Wigner functions

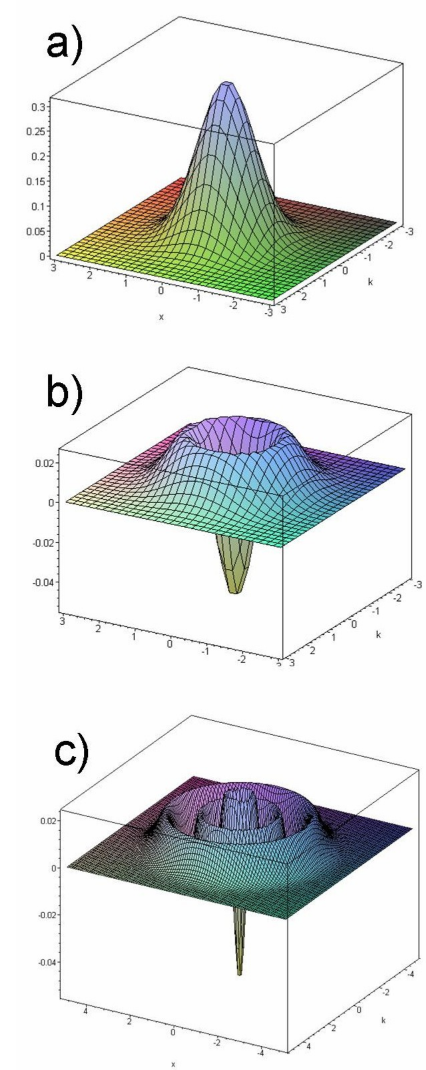
a) Vacuum state
b) Single photon
c) Five photons

Wigner functions

$$W\left(q,p\right) = \frac{1}{2\pi}\int \mathrm{d}x e^{ipx} \left\langle q - \frac{x}{2}\right|_q \hat{\rho} \left| q + \frac{x}{2}\right\rangle_q$$

$$\Rightarrow \left\langle \hat{O} \right\rangle = \mathrm{Tr}\left[\hat{\rho}\hat{O}\right] = \int \mathrm{d}q\mathrm{d}p W_\rho\left(q,p\right) W_O\left(q,p\right)$$

- **May be negative** (e.g. for single photons)
- **Gaussian states**: Gaussian Wigner function
- For pure states, non-negative $\Leftrightarrow$ Gaussian
- **Gaussian transformations**: preserve the Gaussian character of W.f. (modes changes)



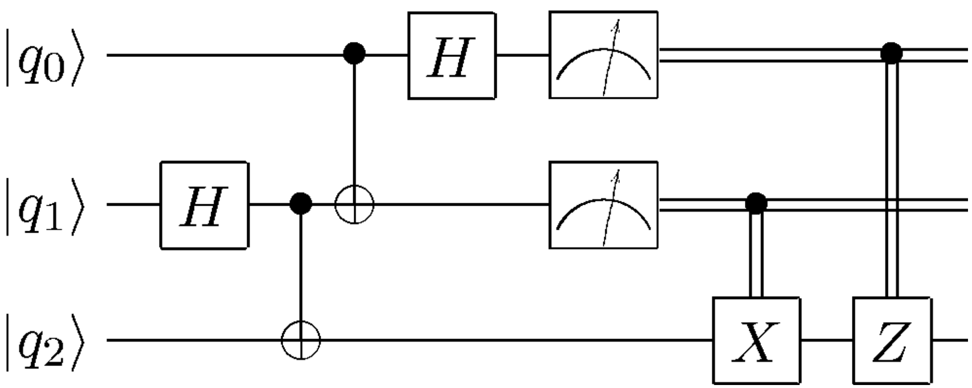
Wigner functions

a) Vacuum state
b) Single photon
c) Five photons

Quantum Computing

- Quantum computation: $$U_f \left|\vec{x}\right\rangle \left|\vec{0}\right\rangle = \left|\vec{x}\right\rangle \left|f\left(\vec{x}\right)\right\rangle$$

- Circuit model:



- Universal set: $$\{H, R_Z(\delta), C_{\mathrm{NOT}}\}$$

QC with continuous variables

- Encode information in CV systems

QC with continuous variables

- Encode information in CV systems
- Define **universal set**:

For any evolution with a hamiltonian **polynomial hamiltonians in the quadratures**

$$\left\{ e^{i\hat{q}s}, e^{i\hat{q}^2 s}, e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, e^{i\hat{q}_1 \otimes \hat{q}_2}, \boxed{e^{i\hat{q}^3 s}} \right\}$$

Single-mode, Gaussian

Non-Gaussian

Two-mode C_Z

S. Lloyd and S. L. Braunstein
Phys. Rev. Lett. 82, 1784 (1999)

QC with continuous variables

- Encode information in CV systems
- Define **universal set**:

  For any evolution with a hamiltonian **polynomial hamiltonians in the quadratures**

$$\left\{ e^{i\hat{q}s}, e^{i\hat{q}^2 s}, e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, e^{i\hat{q}_1\otimes\hat{q}_2}, \boxed{e^{i\hat{q}^3 s}} \right\}$$

Single-mode, Gaussian

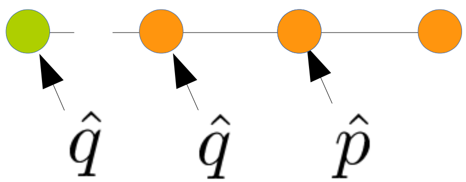
Non-Gaussian

Two-mode C_Z

S. Lloyd and S. L. Braunstein
Phys. Rev. Lett. 82, 1784 (1999)

$$\mathrm{e}^{i\hat{A}\delta t} e^{i\hat{B}\delta t} e^{-i\hat{A}\delta t} e^{-i\hat{B}\delta t} = e^{\left(\hat{A}\hat{B}-\hat{B}\hat{A}\right)\delta t^2} + O(\delta t^3)$$

Measurement-based QC



M. Gu et al,
Phys. Rev. A 79, 62318 (2009)

- The input is coupled with a *special* entangled state – **Cluster**
- All modes are **measured** except for the ones encoding the result
- At each measurement, the state of the mode is transferred to the neighbour modulo some **unitary**
- Choice of the **measurement base** determines the unitary
- Need **adaptive** measurements
- **Homodyne** measurements induce Gaussian transformations

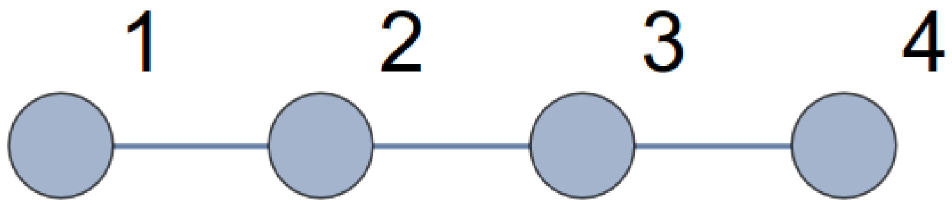
NB: Only **local** measurements are required

Cluster states

- Ideal states (**not physical**)

$$\exp\left(i\sum_{i>j} V_{ij}\hat{q}_i \otimes \hat{q}_j\right)|0\rangle_p^{\otimes N}$$

- Represented as **graphs**
- Eigenstates of **nullifier** operators



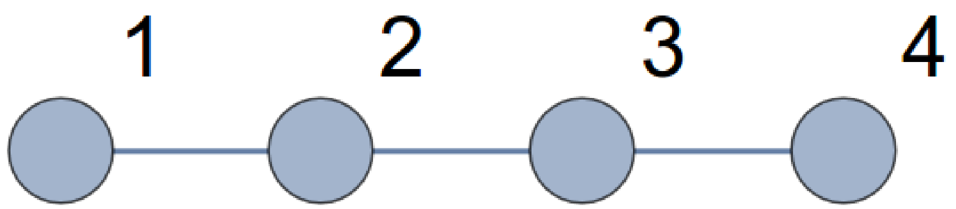
$$\begin{aligned}
\hat{\delta}_1 &= \hat{p}_1 - \hat{q}_2 \\
\hat{\delta}_2 &= \hat{p}_2 - \hat{q}_1 - \hat{q}_3 \\
\hat{\delta}_3 &= \hat{p}_3 - \hat{q}_2 - \hat{q}_4 \\
\hat{\delta}_4 &= \hat{p}_4 - \hat{q}_3
\end{aligned}$$

Cluster states

- Ideal states (**not physical**)

$$\exp\left(i\sum_{i>j} V_{ij}\hat{q}_i \otimes \hat{q}_j\right)|0\rangle_p^{\otimes N}$$

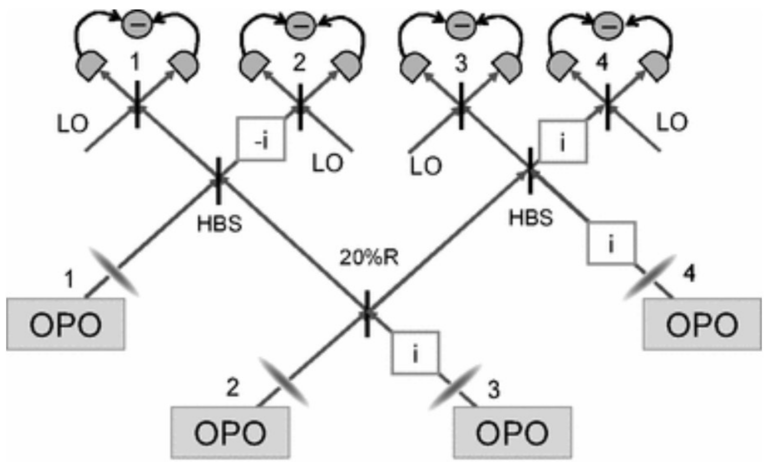
- Represented as **graphs**
- Eigenstates of **nullifier** operators



$$\begin{aligned}\hat{\delta}_1 &= \hat{p}_1 - \hat{q}_2\\ \hat{\delta}_2 &= \hat{p}_2 - \hat{q}_1 - \hat{q}_3\\ \hat{\delta}_3 &= \hat{p}_3 - \hat{q}_2 - \hat{q}_4\\ \hat{\delta}_4 &= \hat{p}_4 - \hat{q}_3\end{aligned}$$

P. van Loock et al, Phys. Rev. A 76, 032321 (2007)

X. Su et al, PRL 98, 070502 (2007)



M. Yukawa et al, Phys. Rev. A 78, 012301 (2008)

- Physical versions: **squeezed states** + **linear optical network** (passive)
- Finite fluctuations = computation errors

Non-Gaussian gates

- Necessary for quantum advantage

S. D. Bartlett and B. C. Sanders
Phys. Rev. Lett. 88(9), 097904 (2002)

Non-Gaussian gates

- Necessary for quantum advantage *S. D. Bartlett and B. C. Sanders Phys. Rev. Lett. 88(9), 097904 (2002)*

- Any one of them allows for universal QC

  The most widespread in the literature is (**cubic phase gate**) $$D\left(\hat{q}\right) = e^{is\hat{q}^3}$$

- **GKP** protocol: count photons in one mode of a TMS state

D. Gottesman et al. Phys. Rev. A 64, 012310 (2001)

Non-Gaussian gates

- Necessary for quantum advantage *S. D. Bartlett and B. C. Sanders Phys. Rev. Lett. 88(9), 097904 (2002)*

- Any one of them allows for universal QC

  The most widespread in the literature is $D\left(\hat{q}\right) = e^{is\hat{q}^3}$ (**cubic phase gate**)

- **GKP** protocol: count photons in one mode of a TMS state

  D. Gottesman et al. Phys. Rev. A 64, 012310 (2001)

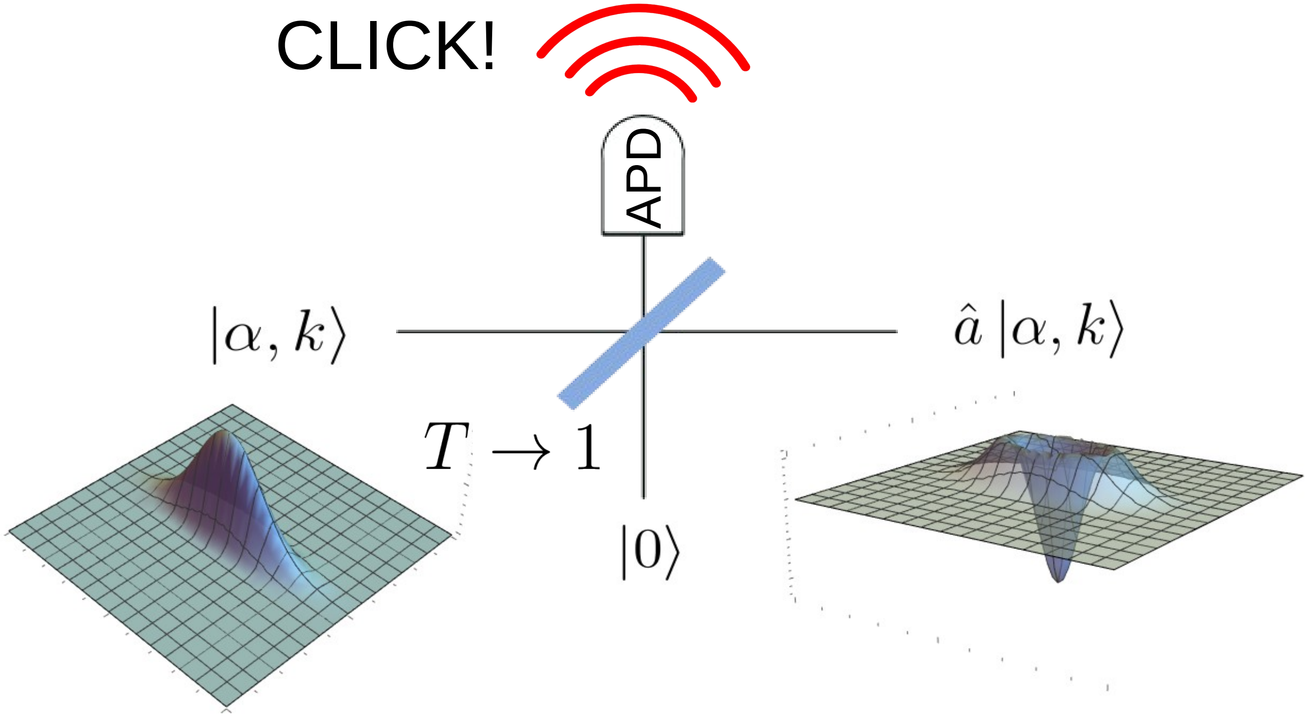
But

- Counting single photons is experimentally challenging

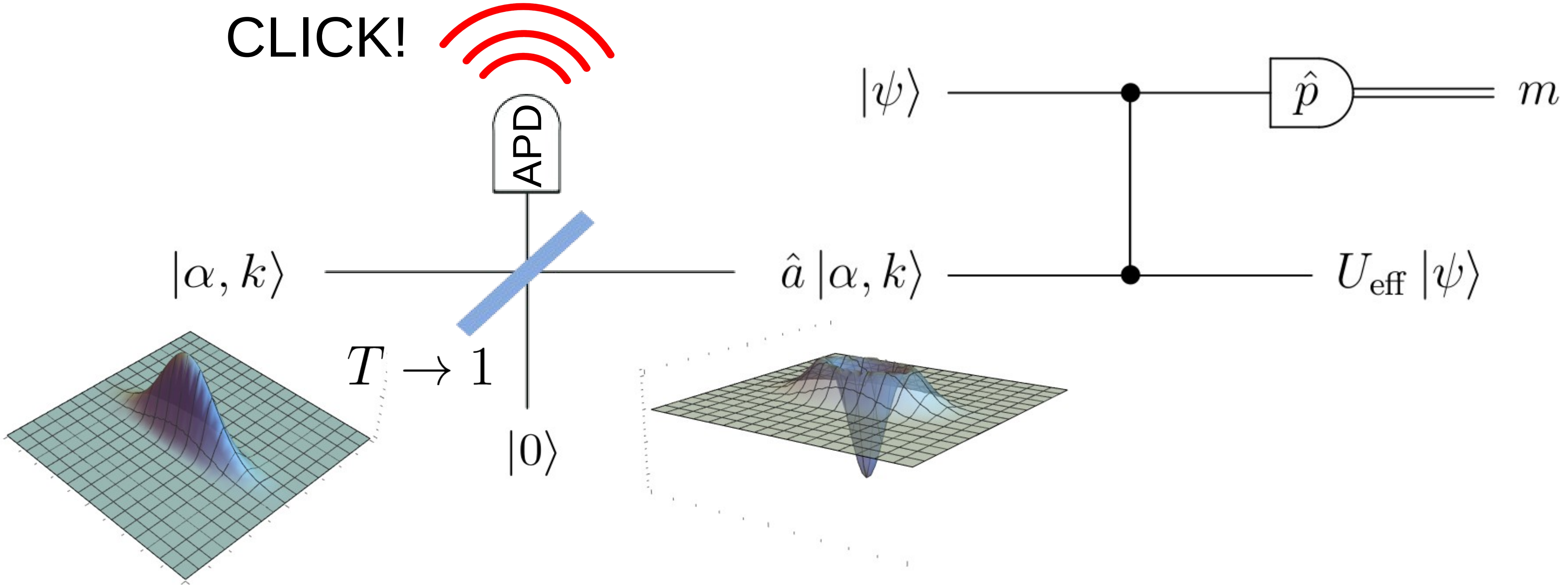
- The required degree of squeezing is a killer

S. Ghose and B. C. Sanders J. Mod. Opt. 54(6), 855–869 (2007)

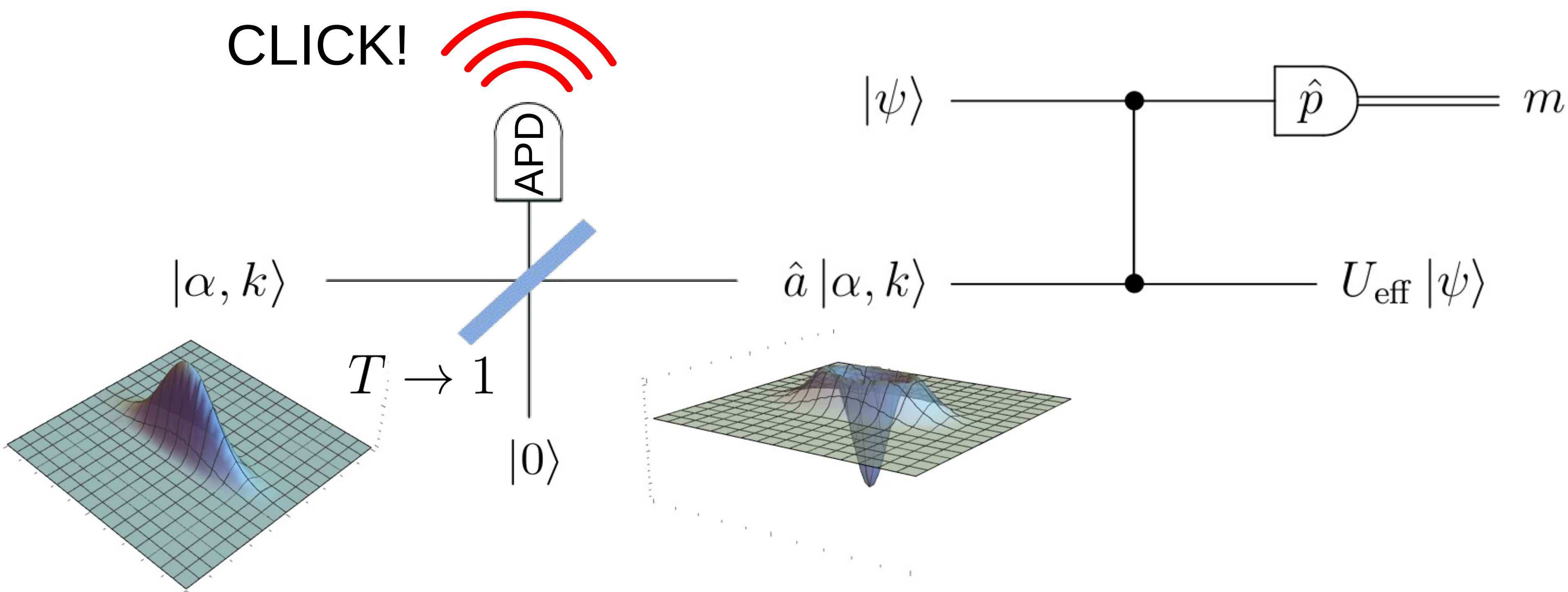
Photon subtraction



Photon subtraction



Photon subtraction



$$U_{\text{eff}} \propto \underbrace{\mathcal{G}[\alpha, k]}_{\text{Displacement}} \underbrace{\exp\left\{-\frac{(\hat{q}+m-\mathrm{Re}[\alpha])^2}{k^2}\right\}}_{\sim\ \text{attenuation}} \underbrace{(\hat{q}+m+\lambda(\alpha, k))}_{\text{Monomial in q}}$$

Polynomial approximation of a unitary operator

- Repeated application: polynomial in the quadratures

P. Marek et al
Phys. Rev. A 84(5), 053802 (2011)

K. Marshall et al
Phys. Rev. A 91, 032321 (2015)

es.

$$e^{is\hat{q}^3} \approx 1 + is\hat{q}^3 = (\lambda - \hat{q})(\eta - \hat{q})(\zeta - \hat{q})$$

- Success probability exponentially drops with the degree
- Deterministic implementation: prepare a resource state offline

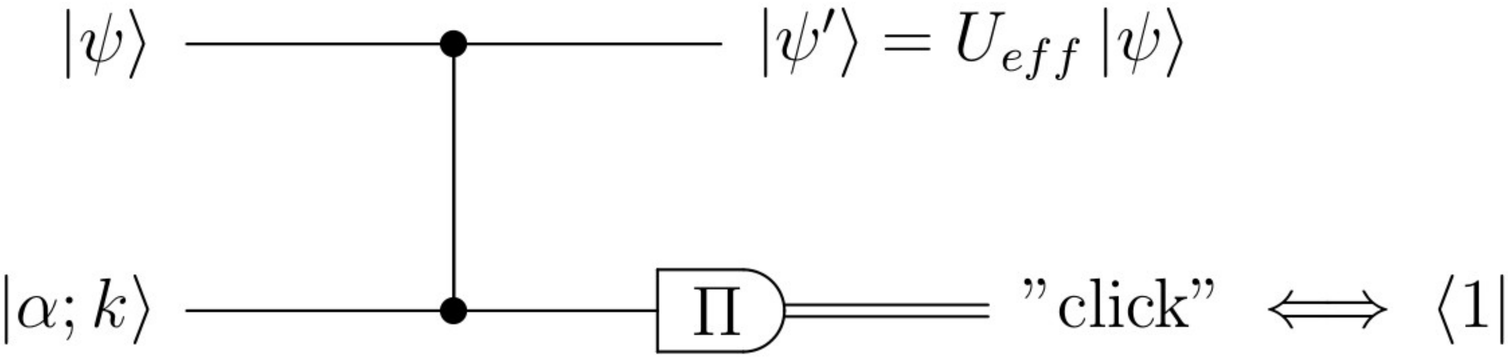
Quality of the gate

$$F_{|\psi\rangle,\vec{m}} = \left|\left(\langle\psi|U_{\text{target}}\right)\left(\mathcal{N}_{\vec{m}}\left(\hat{q}\right)\mathcal{P}_{\vec{m}}\left(\hat{q}\right)|\psi\rangle\right)\right|$$

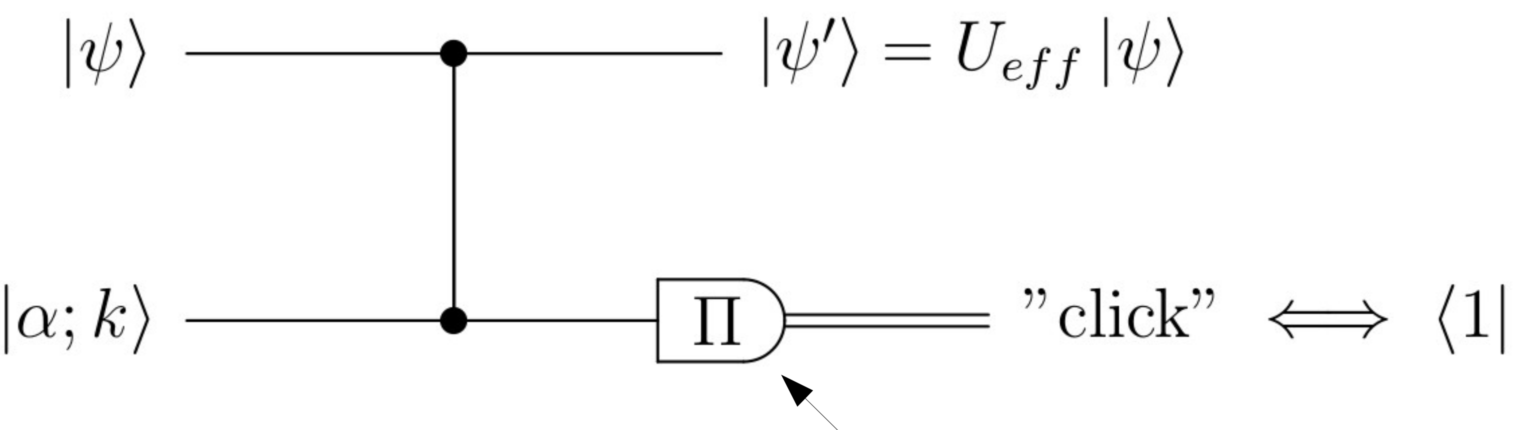
$$\overline{F}_{|\psi\rangle,\delta} = \int_{|\vec{m}|<\delta} p(m) F_{|\psi\rangle,\vec{m}} \mathrm{d}^n m$$

- Depends on the state $p(m) \propto |\psi(m)|^2$
- Trade off: Fidelity - Success probability

Counting one photon

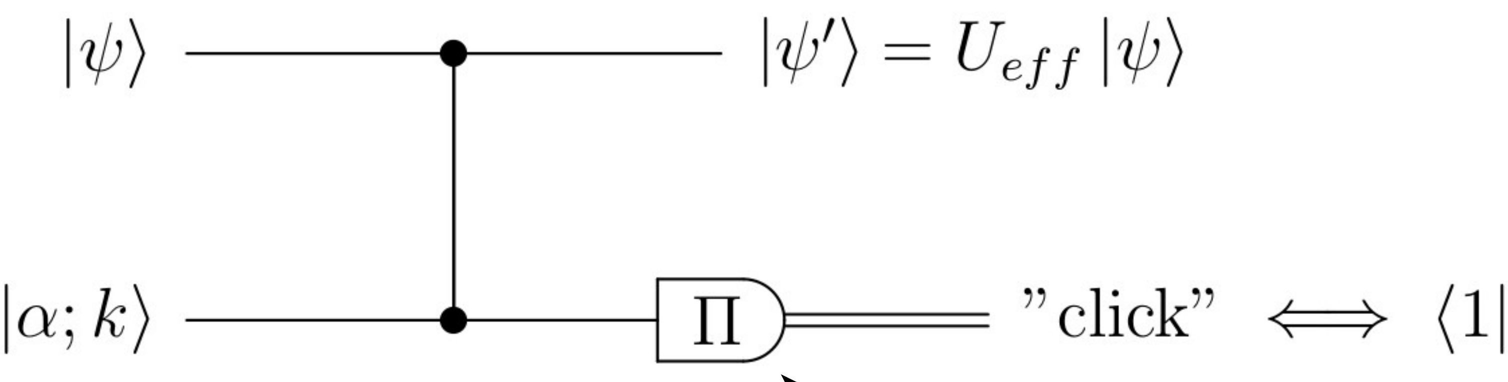


Counting one photon



$$\Pi = \{|0\rangle\langle 0|, |1\rangle\langle 1|, 1 - |0\rangle\langle 0| - |1\rangle\langle 1|\}$$

Counting one photon



$$\Pi = \{|0\rangle\langle 0|, |1\rangle\langle 1|, 1 - |0\rangle\langle 0| - |1\rangle\langle 1|\}$$

$$\mathrm{U_{eff}} \propto \underbrace{\mathcal{G}\left[\alpha, k\right]}_{\text{Displacement}} \underbrace{\exp\left\{-\frac{k^2}{4+k^2}\left(\hat{q} + \frac{\mathrm{Im}[\alpha]}{2}\right)^2\right\}}_{\sim\text{ attenuation}} \underbrace{\left(\hat{q} + \lambda\left(\alpha, k\right)\right)}_{\text{Monomial in q}}$$

Displacement

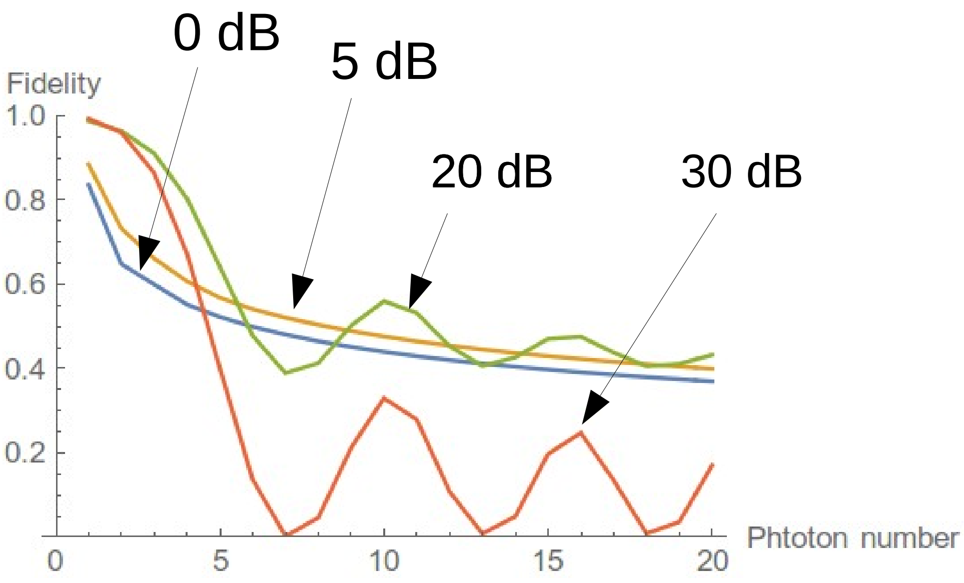
~ attenuation

Monomial in q

(no measurement result)

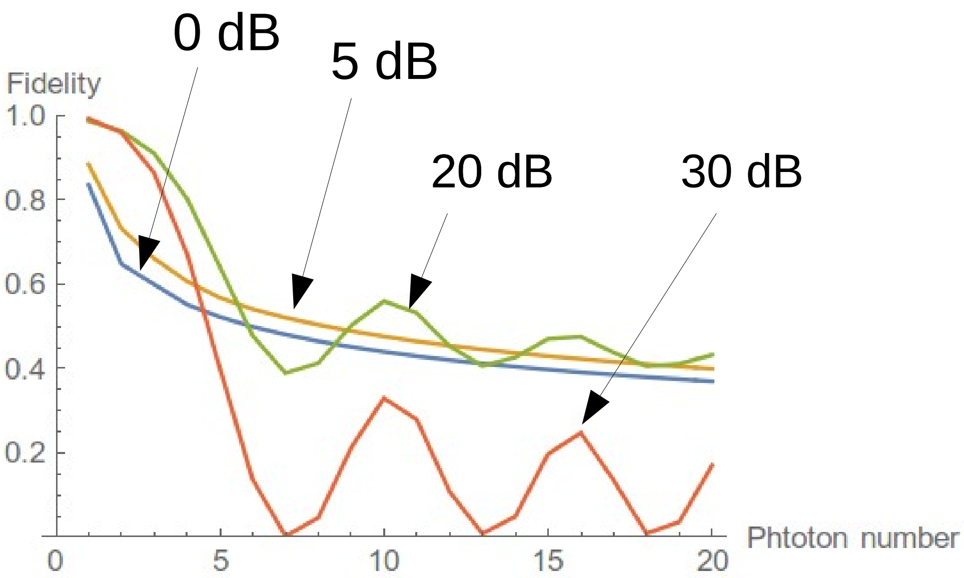
Fidelity

Input states: $|n\rangle$

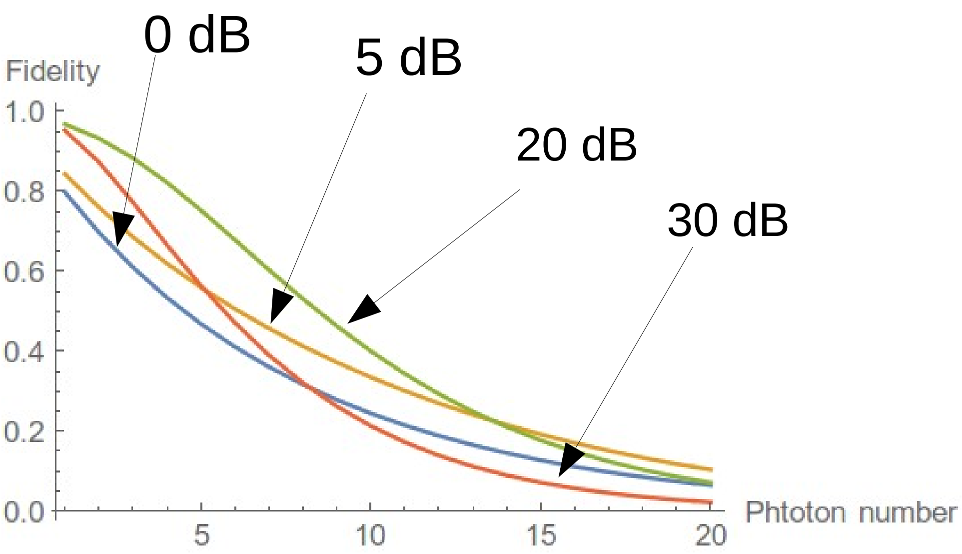


Fidelity

Input states: $|n\rangle$

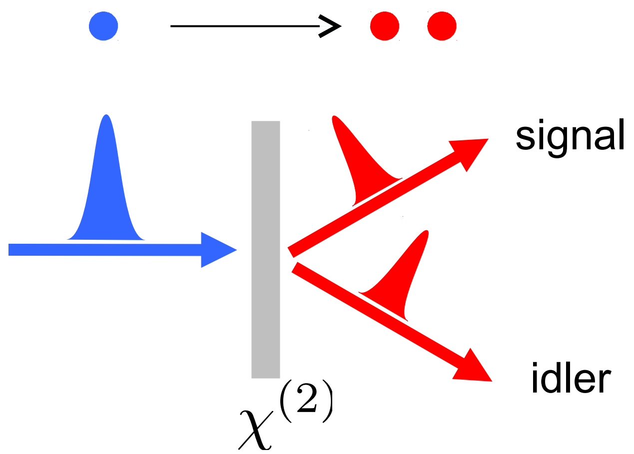


Input states: $|\beta\rangle, \quad \mathrm{Im}\left[\beta\right] = 0$

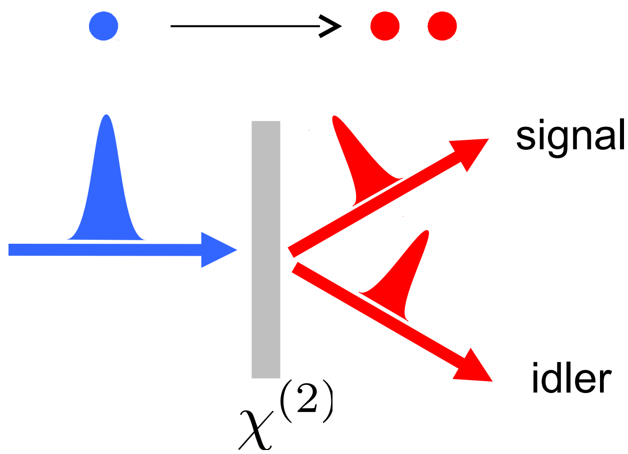


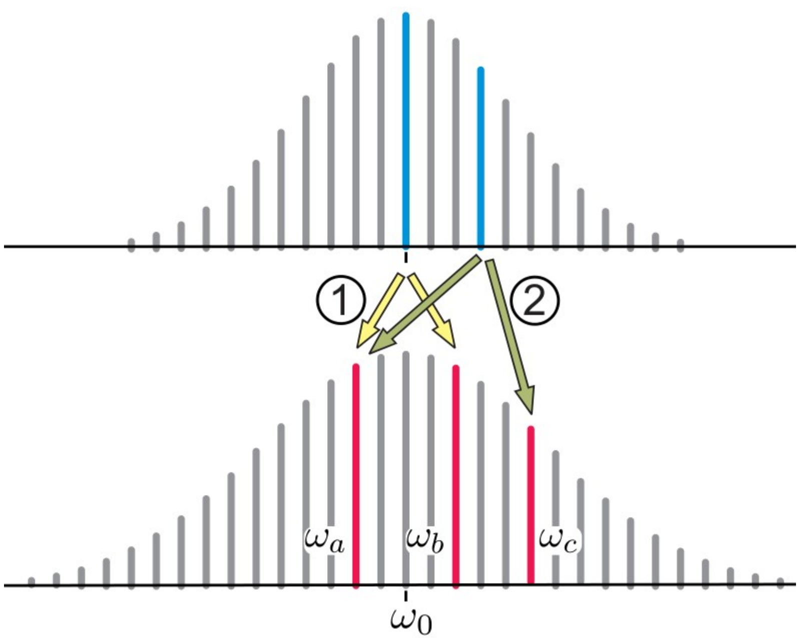
$$\langle \hat{N} \rangle = |\beta|^2$$

Experiments at LKB: Entanglement

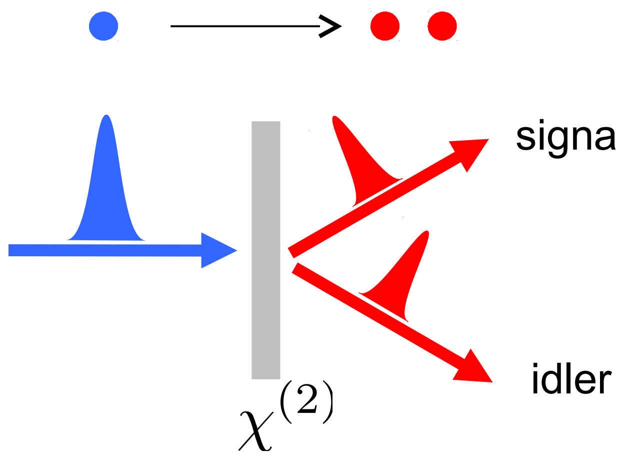


Experiments at LKB: Entanglement



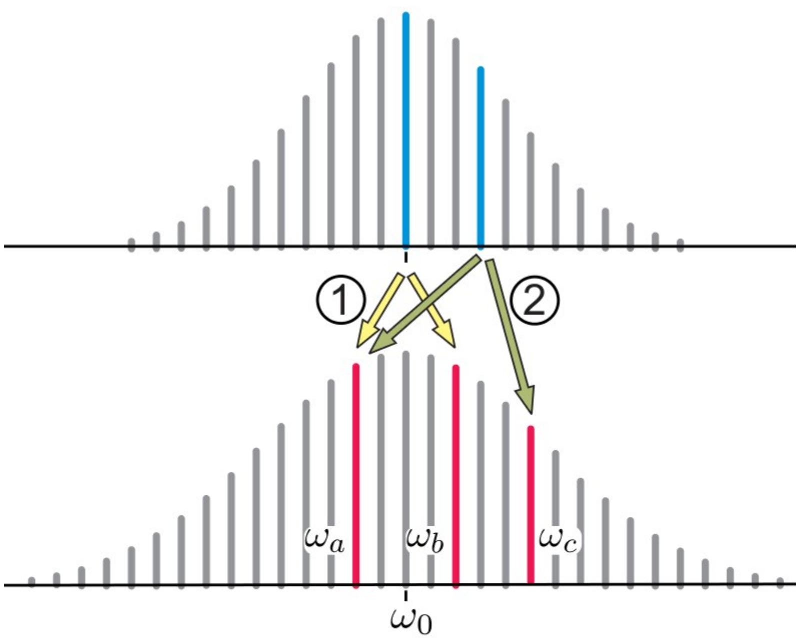


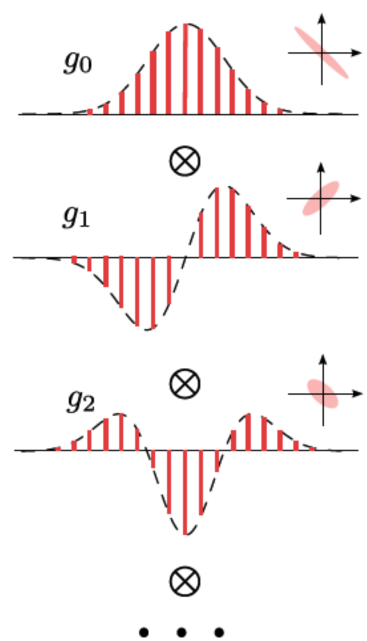
Experiments at LKB: Entanglement



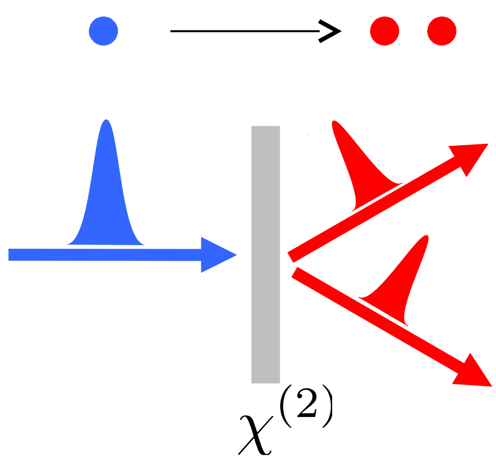
$$
\begin{aligned}
H &= i\hbar g \sum_{m,n} L_{mn} \hat{a}_m^\dagger \hat{a}_n^\dagger + \text{h.c.} \\
&= i\hbar g \sum_{m,n} \Lambda_k \hat{S}^{\dagger 2}_k + \text{h.c.}
\end{aligned}
$$

J. Roslund et al
Nat. Phot. 8, 109-112 (2014)



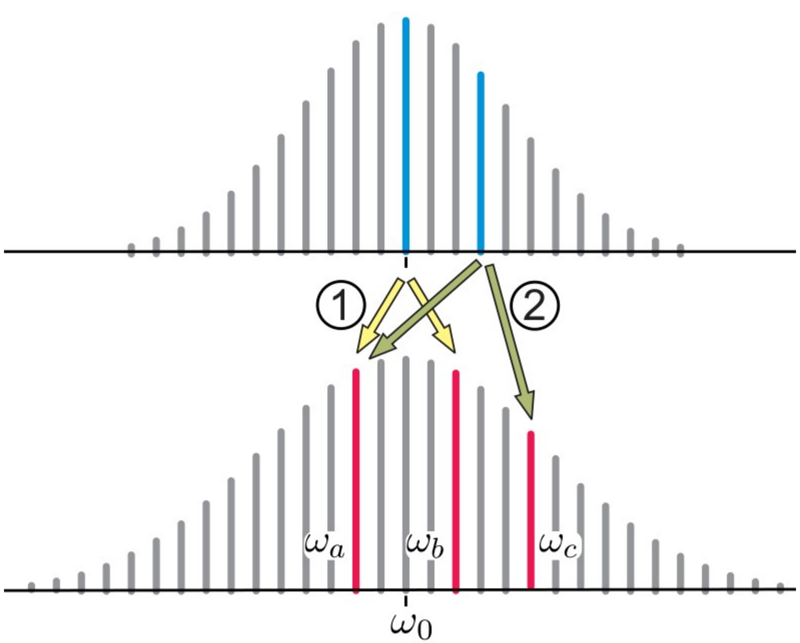


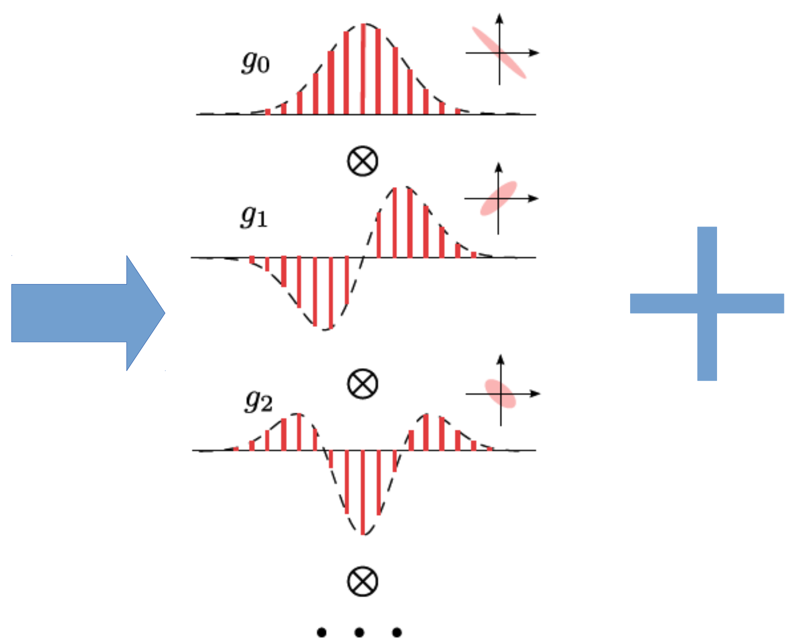
Experiments at LKB: Entanglement



$$
\begin{aligned}
H &= i\hbar g \sum_{m,n} L_{mn} \hat{a}_m^\dagger \hat{a}_n^\dagger + \text{h.c.} \\
&= i\hbar g \sum_{m,n} \Lambda_k \hat{S}_k^{\dagger 2} + \text{h.c.}
\end{aligned}
$$

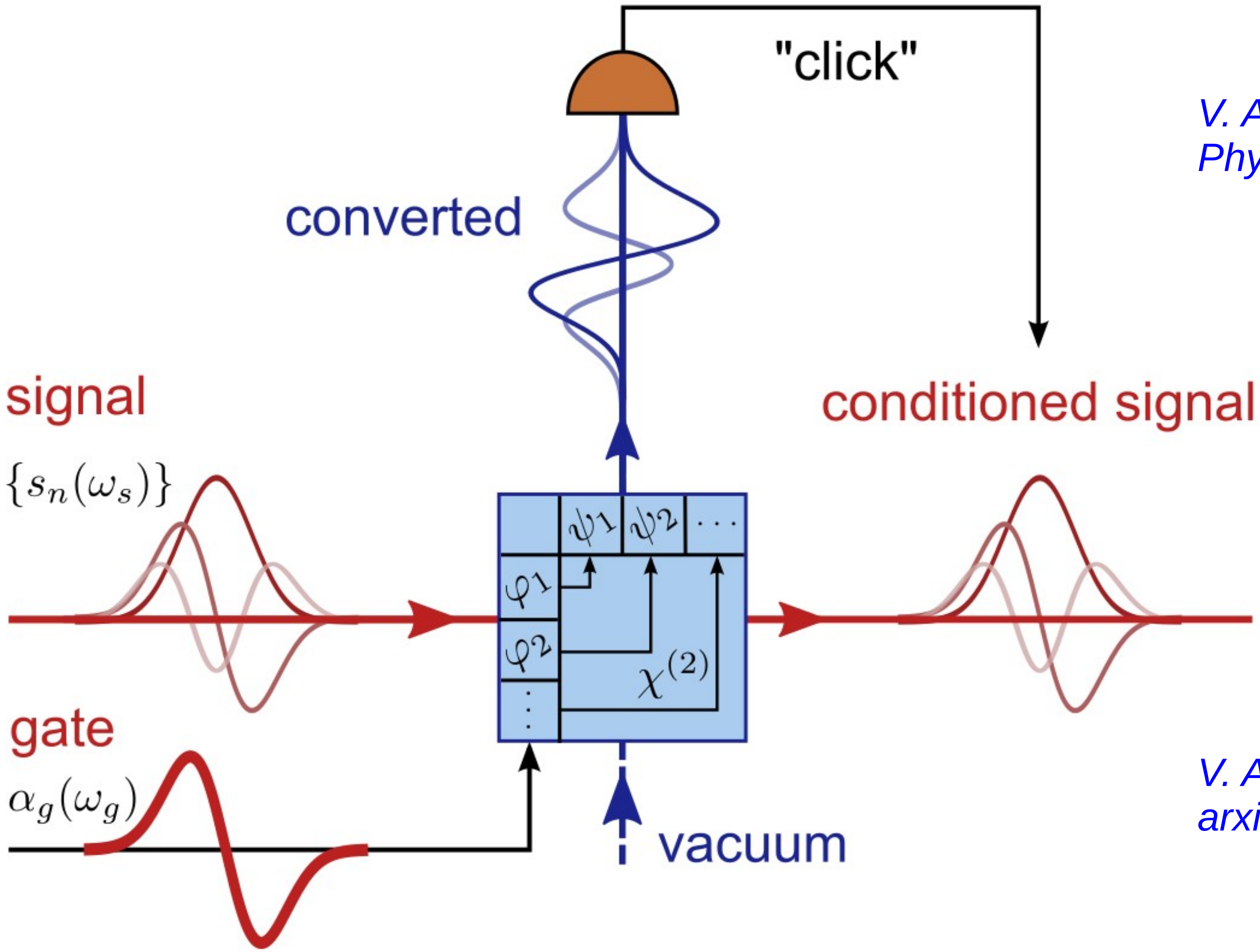
J. Roslund et al
Nat. Phot. 8, 109-112 (2014)





Spectrally resolved detection

Experiments at LKB: Single photon operations



V. Averchenko et al
Phys. Rev. A 89, 063808 (2014)

V. Averchenko et al
arxiv:1510.01198 (2015)

THANK YOU!!!