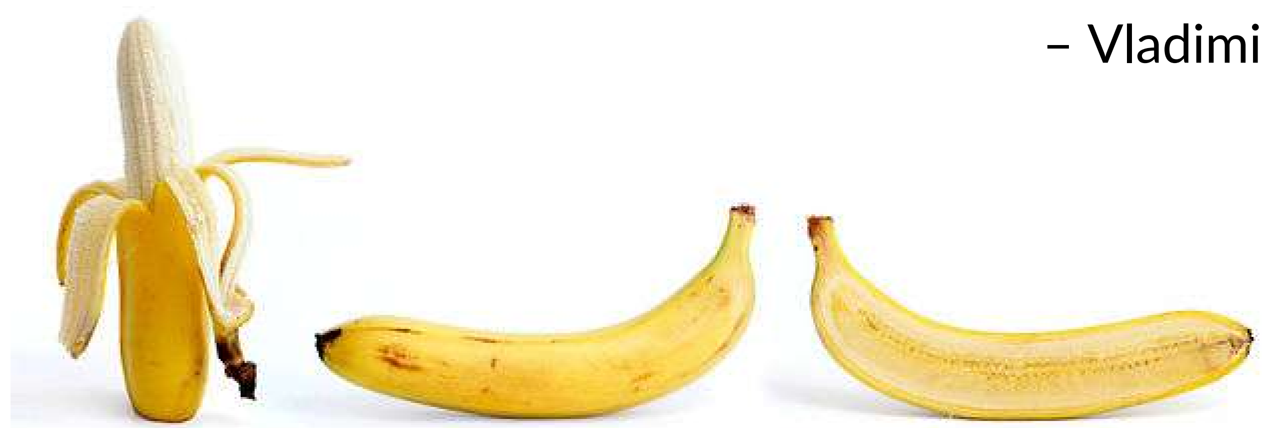
Gaussian and non-Gaussian resources in quantum optics

Francesco Arzani

quantum resources **Gaussian** **non-Gaussian**

Classification of ~~mathematical problems~~ as ~~linear~~ and ~~nonlinear~~ is like classification of the Universe as bananas and non-bananas.

– Vladimir Arnol'd (?)



Inria

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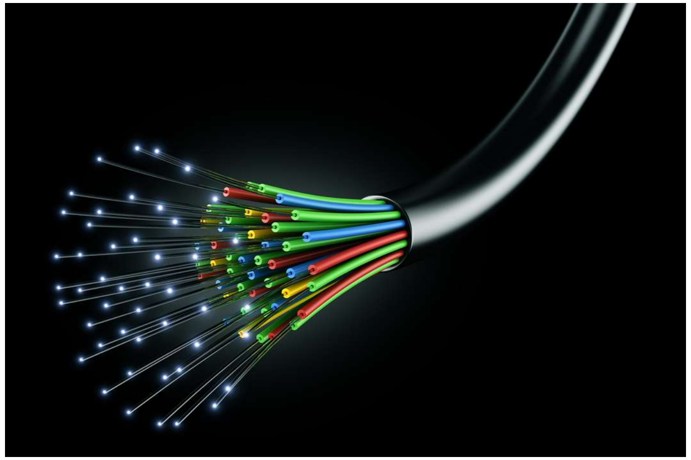
Quantum
Architectures,
Algorithms,
Applications
and their **T**heory

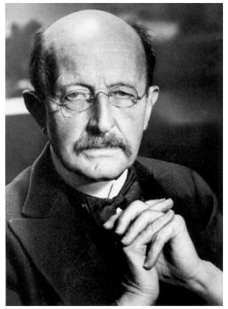
Quantum optics

Light is used for ordinary (classical) communications

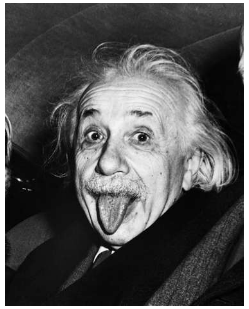


Light had a central role in the development of Quantum Mechanics

Atomic spectra



Blackbody radiation



Photoelectric Effect

Quantum Electrodynamics

⇨ Lots of know-how, theory, instrumentation

Photons interact weakly ⇨ Easy to protect fragile quantum states

Quantum information processing

Bits : $\psi \in \{0, 1\}$

Qubits : finite dimension → discrete variables

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Continuous variables : infinite dimension

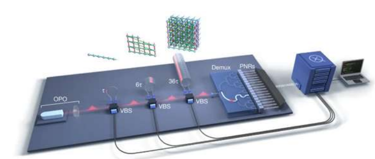
Also known as *qu-modes*, *bosonic systems*

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + \dots$$

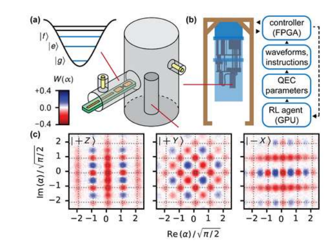
- More general
- Large scale entanglement...

XANADU ALICE & BOB QUANDELA aws

Computational advantage [MLF+22]



Error correction [SER+22]



[MLF+22] Nature 606 (2022)
[SER+22] arXiv:2211.09116 (2022)

Outline

1. Continuous variables in quantum optics : an introduction
2. Gaussian operations : definitions, experimental tools
3. Protocols accessible through Gaussian operations
4. Limitations of Gaussian operations
5. Non-Gaussian resources in optics
6. Protocols accessible with non-Gaussian operations

Formalism

Rosetta stone

DV : information encoded in *d*-level systems (typically *d* = 2)

$$\alpha\,|0\rangle + \beta\,|1\rangle$$

$$\Pr(0) = |\alpha|^2$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$\mathcal{H} = \mathbb{C}^2$$

CV : information encoded in observables with continuous spectrum, e.g. : $\hat{q}$, $\hat{p}$

$$\int_{\mathbb{R}} \psi(x)\,|x\rangle_q\,\mathrm{d}x$$

$$\Pr(q \in [x, x + \mathrm{d}x]) = |\psi(x)|^2\,\mathrm{d}x$$

$$\int_{\mathbb{R}} |\psi(x)|^2\,\mathrm{d}x = 1$$

$$\mathcal{H} = L^2(\mathbb{R}, \mathbb{C})$$

Continuous-variable quantum systems

Free electromagnetic field

$$H = \frac{\epsilon_0}{2}\int_\Omega d^3r\left[\boldsymbol{E}^2(\boldsymbol{r},t) + c^2\boldsymbol{B}^2(\boldsymbol{r},t)\right]$$

$$= \frac{1}{2}\sum_j \omega_j\left(Q_j^2(t) + P_j^2(t)\right)$$

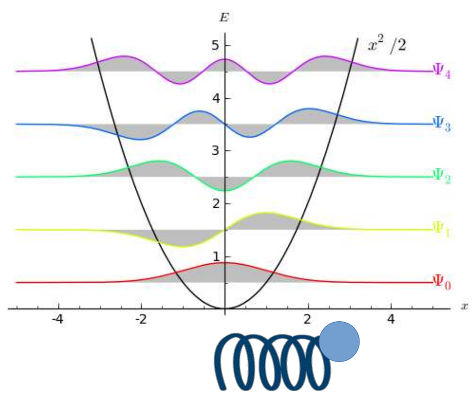
Expand on basis Maxwell Eqs. solutions

Impose: $\left[\hat{Q}_j, \hat{P}_l\right] = i\delta_{j,l}$

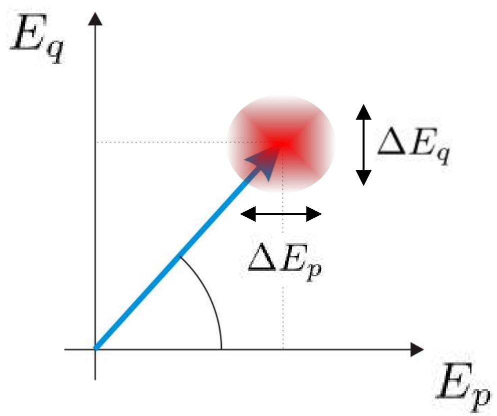
Define: $\hat{a}_j = \frac{\hat{Q}_j + i\hat{P}_j}{\sqrt{2}}$

$$\hat{H} = \sum_j \omega_j\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

Abstraction: harmonic oscillators



Physical realization: electric field



$$\hat{E}_Q \propto \hat{a} + \hat{a}^\dagger$$

$$\hat{E}_P \propto i(\hat{a}^\dagger - \hat{a})$$

Phase space description

Quadratures ~ position and momentum ⇨ Phase space representation

Wigner function ~ Quasi-probability distribution in phase space

$$W_\rho(q,p) = \frac{1}{2\pi}\int dx e^{ipx}\langle q - x/2|\rho|q + x/2\rangle$$

Marginals

$$\mathrm{Pr}(q) = \int \mathrm{d}p W_\rho(q,p)$$

$$\mathrm{Pr}(p) = \int \mathrm{d}q W_\rho(q,p)$$

Trace

$$\mathrm{Tr}\left[AB\right] = \int \mathrm{d}q\mathrm{d}p W_A(q,p) W_B(q,p)$$

Phase space description

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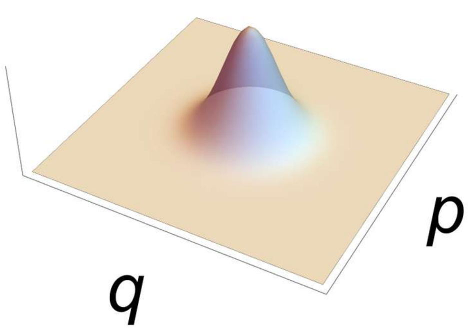
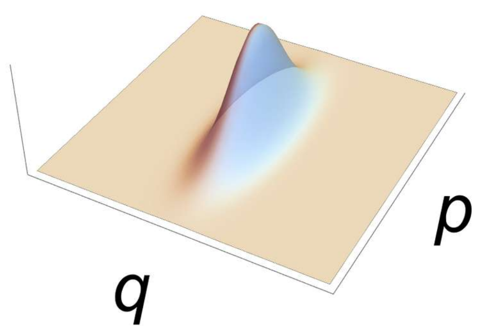
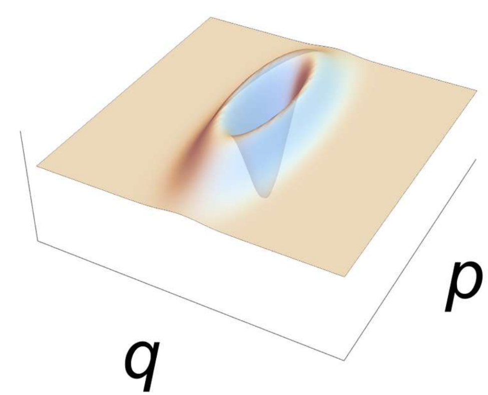
Gaussian states:



Vacuum → Same marginals



Squeezing



May be negative!

Aside: modes

Electric field as many **modes**:

Normalized solutions $f_j\left(\vec{r},t\right)$ of Maxwell's equations

Mode basis

$$\int f_n(\vec{r},t)\cdot f_m^*(\vec{r},t)d^3r dt=\delta_n^m$$

Quantum Hilbert space(s)

$$f_1 \longleftrightarrow \hat{a}_1$$

$$f_2 \longleftrightarrow \hat{a}_2$$

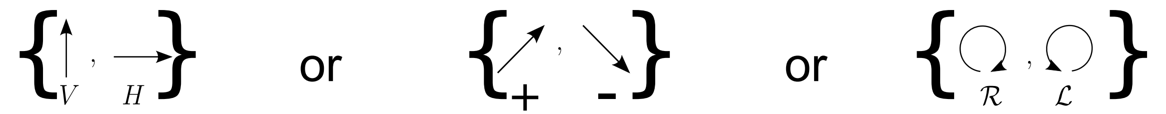
$$\vdots \qquad \vdots$$

$$|\Psi\rangle=\sum_{n1,n2,\ldots} A_{n1n2\ldots}|n1:f_1\rangle\otimes|n2:f_2\rangle\otimes\ldots$$

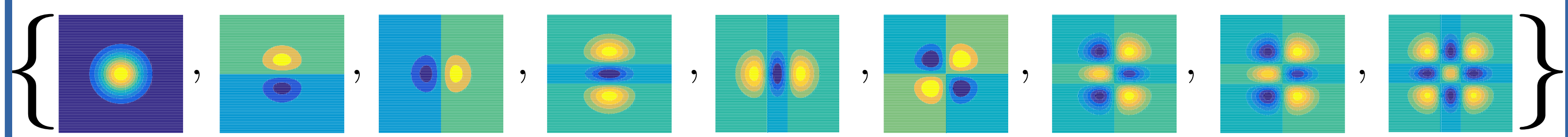
What exactly is a mode?

Normalized solutions $f_j\left(\vec{r}, t\right)$ of Maxwell's equations

Polarization modes



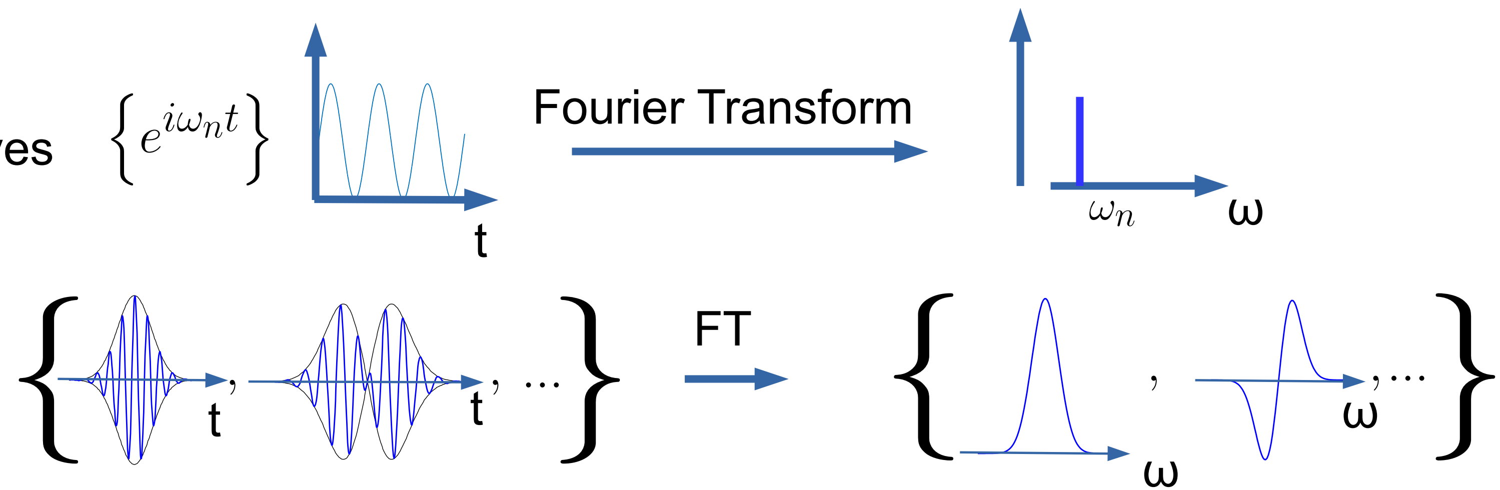
Spatial modes



Temporal modes

Plane waves $\left\{e^{i\omega_n t}\right\}$

HG modes



Gaussian operations

Gaussian states

$$\hat{a}|0\rangle = 0 \qquad \langle\hat{q}\rangle = \langle\hat{p}\rangle = 0 \qquad \Delta^2\hat{q} = \Delta^2\hat{p} = \frac{1}{2}$$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \qquad \begin{gathered}\langle\hat{q}\rangle \propto \mathrm{Re}(\alpha)\\ \langle\hat{p}\rangle \propto \mathrm{Im}(\alpha)\end{gathered} \qquad \Delta^2\hat{q} = \Delta^2\hat{p} = \frac{1}{2}$$

$$\hat{D}(\alpha)\hat{S}(r)|0\rangle \qquad \begin{gathered}\langle\hat{q}\rangle \propto \mathrm{Re}(\alpha)\\ \langle\hat{p}\rangle \propto \mathrm{Im}(\alpha)\end{gathered} \qquad \Delta^2\hat{q} = \frac{1}{\Delta^2\hat{p}} = \frac{e^{-2r}}{2}$$

$$\rho_{\mathrm{th}} \propto e^{-\beta\hat{a}^\dagger\hat{a}} \qquad \langle\hat{q}\rangle = \langle\hat{p}\rangle = 0 \qquad \Delta^2\hat{q} = \Delta^2\hat{p} \geq \frac{1}{2}$$

Gaussian unitary operators

Vector notation

$$\boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{p} \end{pmatrix} \quad [\xi_j, \xi_k] = iJ_{jk}$$

$$J = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}$$

Standard symplectic form

$$U_G^\dagger \boldsymbol{\xi} U_G = S\boldsymbol{\xi} + \boldsymbol{x} = \boldsymbol{\xi}'$$

Phase-space deformation

Phase-space translation

Generated by quadratic polynomials:

$$U_G = \exp\left[-it\mathcal{P}\left(\hat{q},\hat{p}\right)\right]$$

$$\deg\left(\mathcal{P}\right) \leq 2$$

$$[\xi'_j, \xi'_k] = iJ_{jk} \iff S^T J S = J$$

$$\mathrm{Sp}\left(2n, \mathbb{R}\right)$$

Action on Wigner function:

$$W_{U_G \rho U_G^\dagger}(\boldsymbol{x}) = W_\rho(S^{-1}(\boldsymbol{x} - \boldsymbol{\eta}))$$

A. B. Dutta, N. Mukunda, R. Simon, Pramana 45 (1995)

Gaussian unitary operators

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$$\mathrm{Sp}\,(2n, \mathbb{R})$$

Bloch-Messiah (Euler) decomposition:

$$S = R_1 K R_2$$

Squeezing:

$$K = \mathrm{diag}\left(e^{r_1}, \ldots, e^{r_n}, e^{-r_1}, \ldots, e^{-r_n}\right)$$

Linear optics (passive **interferometers**):

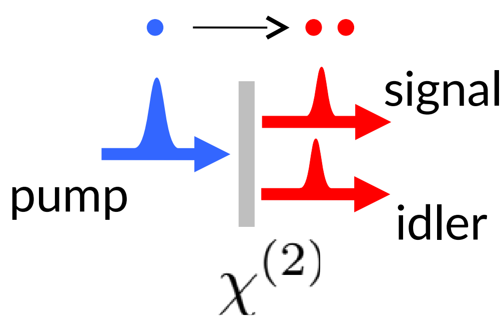
$$R = \begin{pmatrix} X & -Y \\ Y & X \end{pmatrix}, \quad X + iY \in U(n)$$

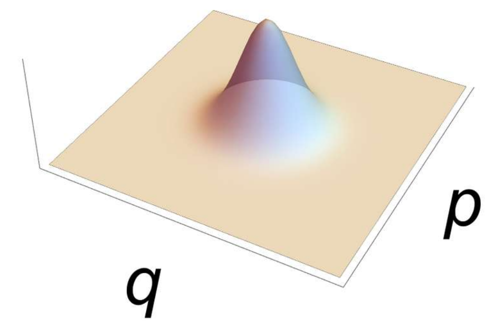
A. B. Dutta, N. Mukunda, R. Simon, Pramana 45 (1995)

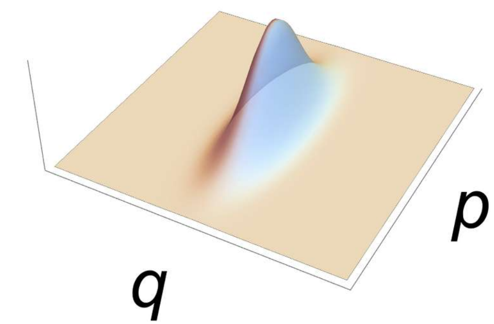
Squeezing

Heisenberg picture:

$$\begin{pmatrix} q^{(0)} \\ p^{(0)} \end{pmatrix} \mapsto \begin{pmatrix} e^{-r} q^{(0)} \\ e^{r} p^{(0)} \end{pmatrix}$$







$$\lim_{r\to\infty} |r\rangle = |0\rangle_q$$

$$\lim_{r\to-\infty} |r\rangle = |0\rangle_p$$

$$|r\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{n=0}^{\infty} (-\tanh r)^n \frac{\sqrt{(2n)!}}{2^n n!} |2n\rangle$$

Linear optics

- Conserve total particle number (« passive »)
- Symplectic & orthogonal action
- Can be constructed as beam splitters + free evolution

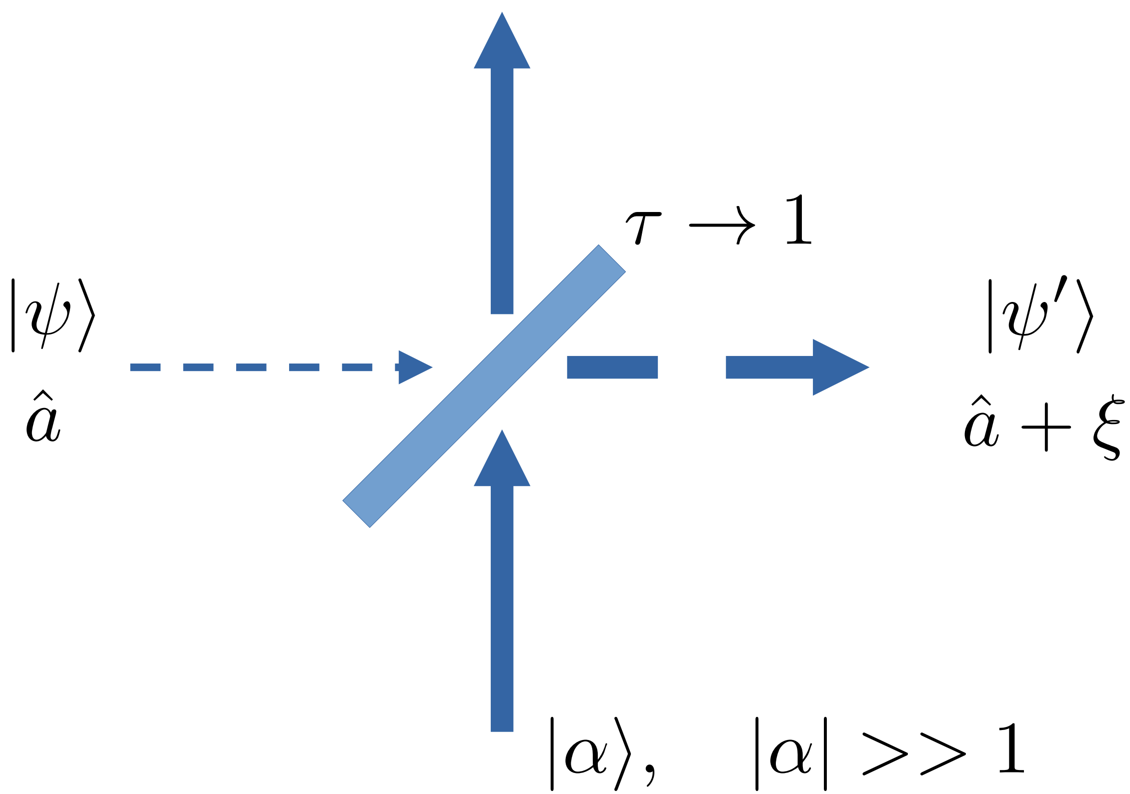
Change of modes ⟷ **Linear optics**

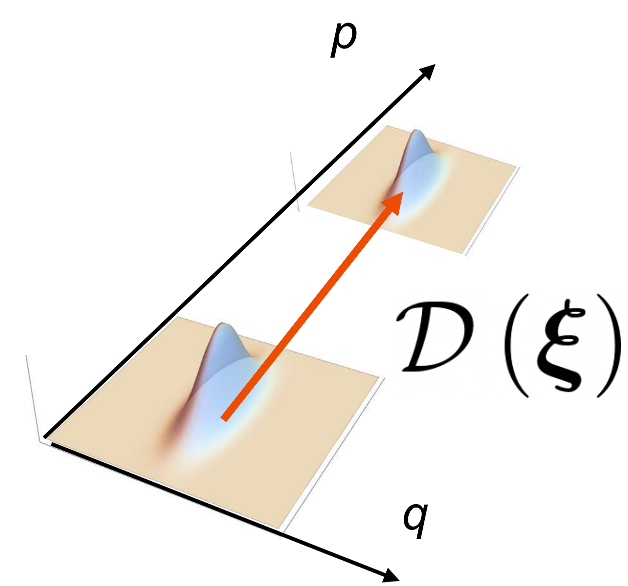
$$g_k(\vec{r},t) = \sum_m U_{km} f_m(\vec{r},t)$$

$$\hat{b}_k = \sum_m \left(U^\dagger\right)_{km} \hat{a}_m$$

Displacements

Change mean value

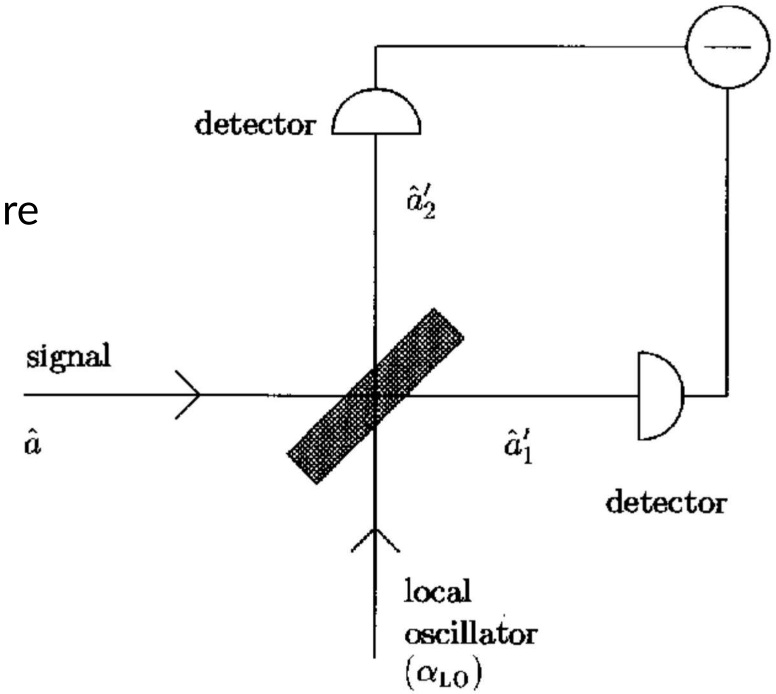




Measurements

Homodyne detection

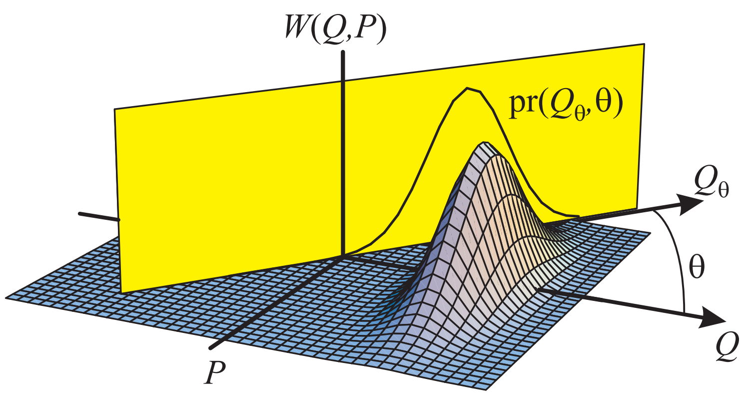
Phase of LO → measured quadrature



$$\alpha_{\mathrm{LO}} = |\alpha_{\mathrm{LO}}| e^{i\theta}$$

$$\hat{q}_\theta = \cos\theta \hat{q} - \sin\theta \hat{p}$$

==> Tomography

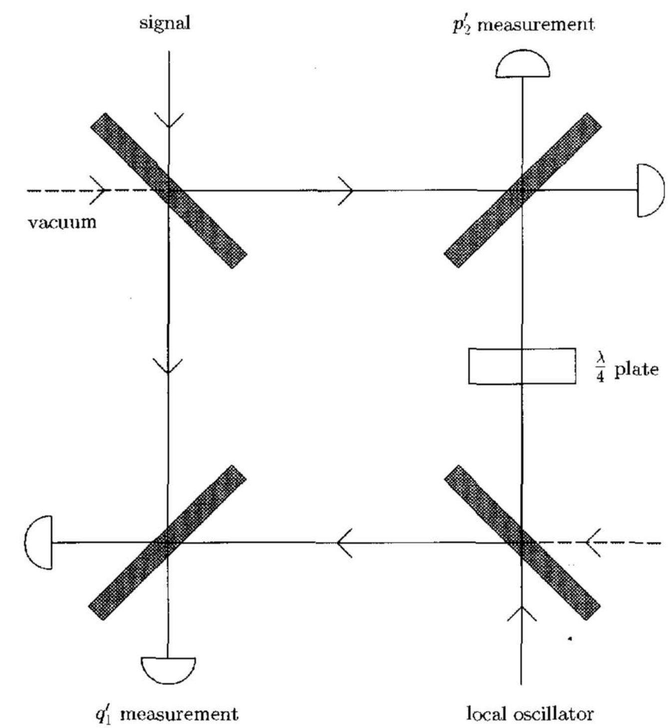


A.I.Lvovsky, M.G.Raymer, Rev. Mod. Phys. 81 (2009)

U. Leonhardt, Cambridge U. P. (1997)

Heterodyne detection

Projects on coherent state

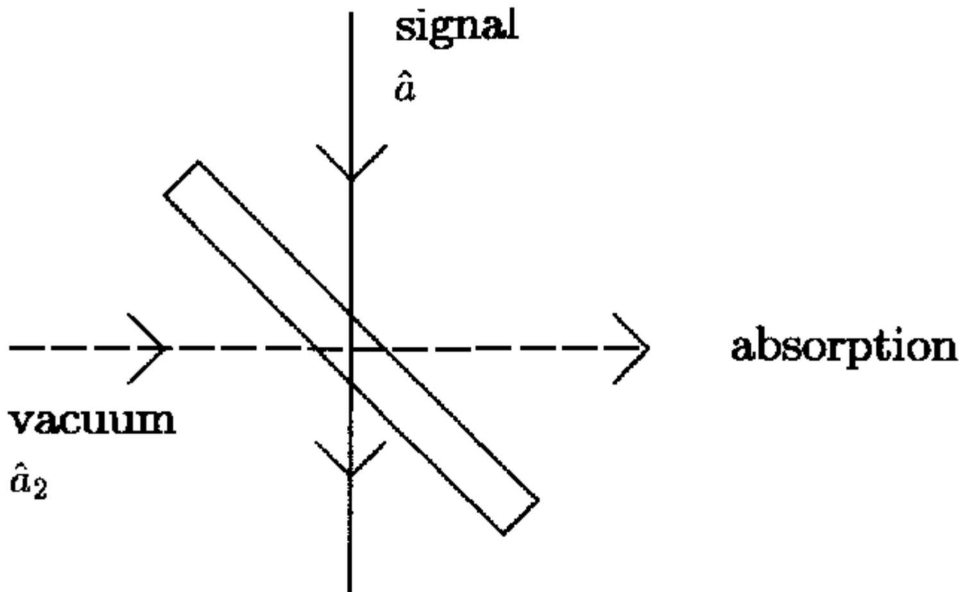


Channels

Stinespring:

1. Add auxiliary systems in Gaussian states
2. Perform overall Gaussian unitary
3. Measure some modes
4. Discard some modes

Example: photon loss



U. Leonhardt,
Cambridge U. P. (1997)

Protocols using only Gaussian operations

Gaussian quantum protocols

Quantum teleportation

Quantum key distribution

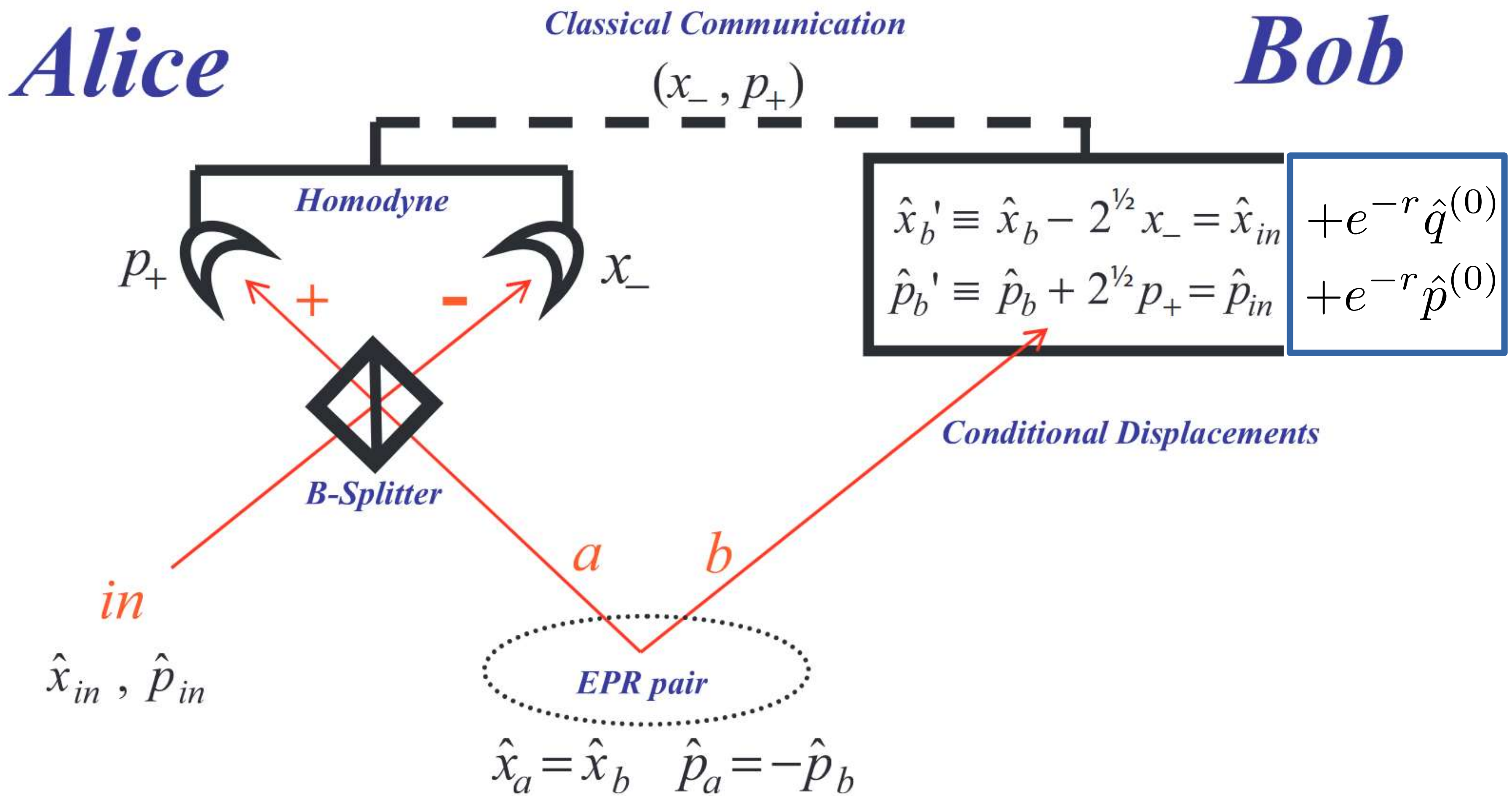
Quantum illumination

Quantum secret sharing

Quantum error correction*

...

Quantum teleportation



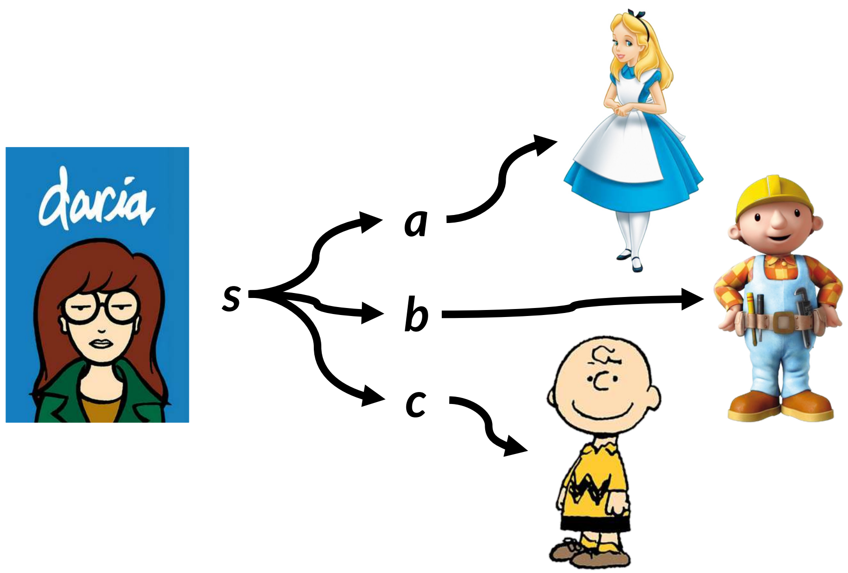
$$U_{\text{BS}}|r\rangle|-r\rangle \to U_{\text{BS}}|0\rangle_q|0\rangle_p$$

Quantum cryptography : Quantum state sharing

Multi-party cryptographic primitive
to securely share a quantum state

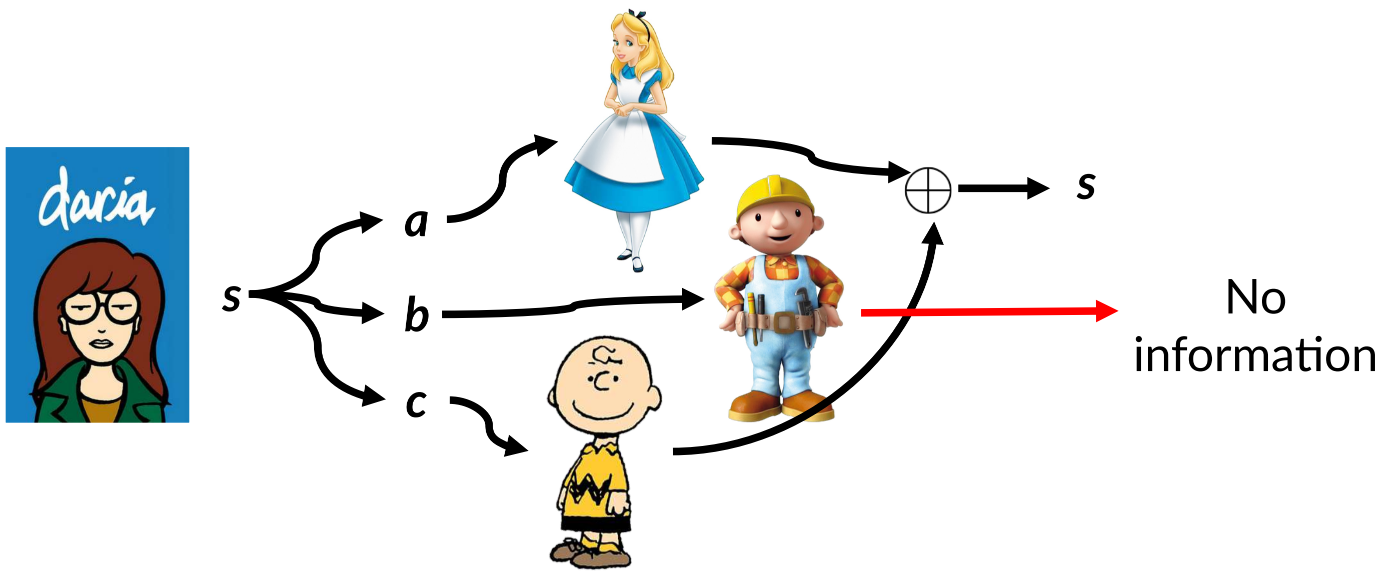
Quantum cryptography : Quantum state sharing

Multi-party cryptographic primitive
to securely share a quantum state



Quantum cryptography : Quantum state sharing

Multi-party cryptographic primitive to securely share a quantum state



Quantum cryptography : Quantum state sharing

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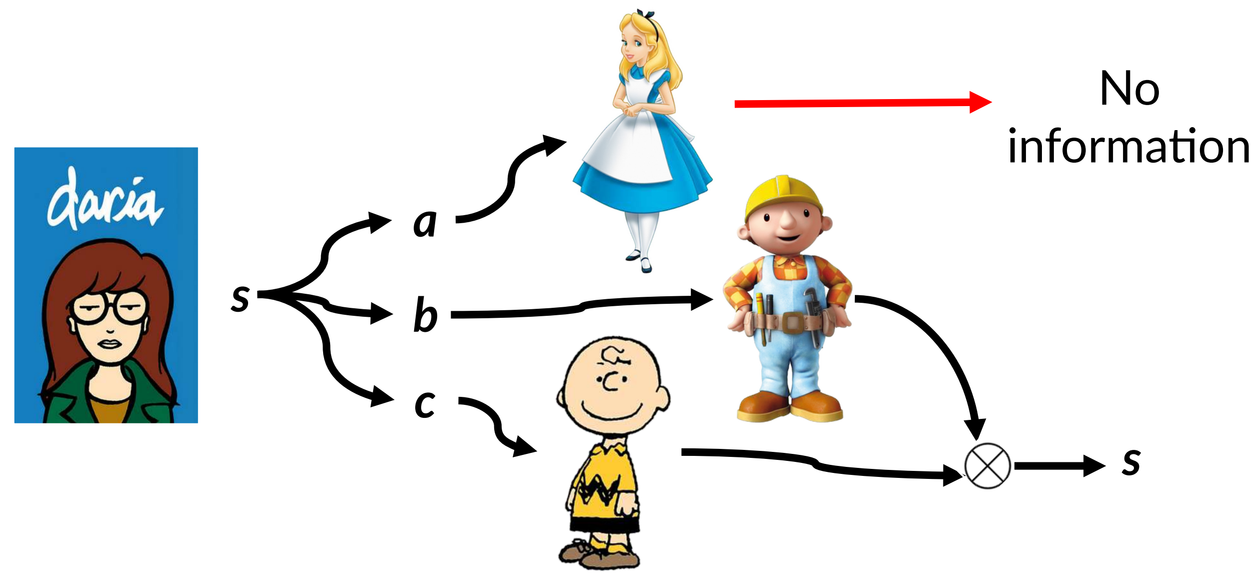
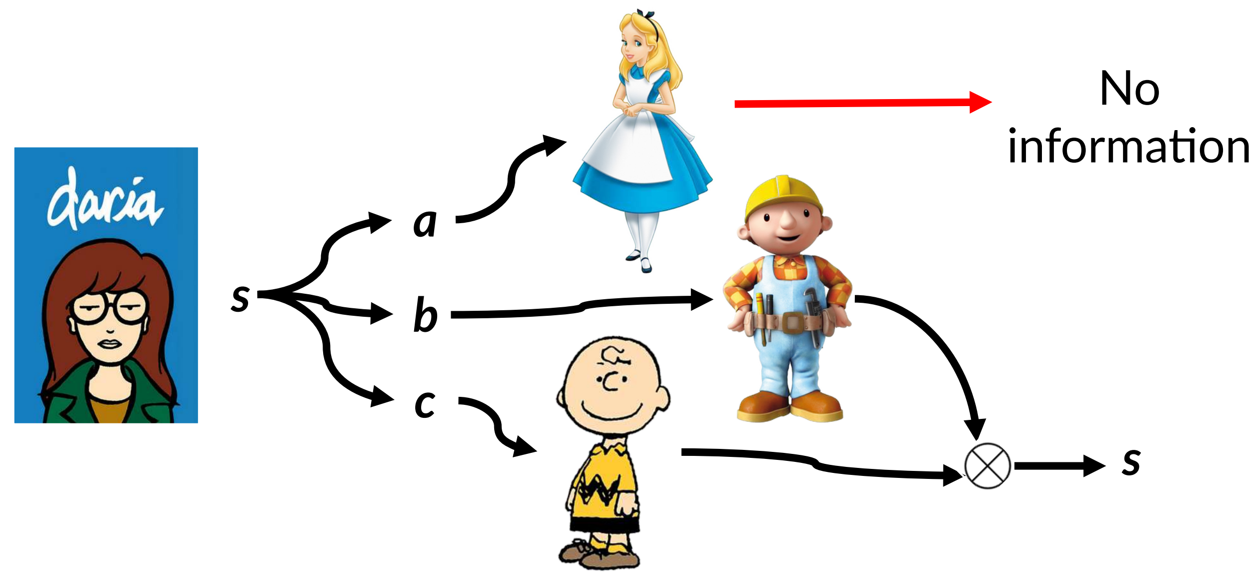


Francesco Arzani

Quantum cryptography : Quantum state sharing

Multi-party cryptographic primitive
to securely share a quantum state



Theory

D. Markham & P. van Loock
AIP Conference Proceedings (2011)

Experiments

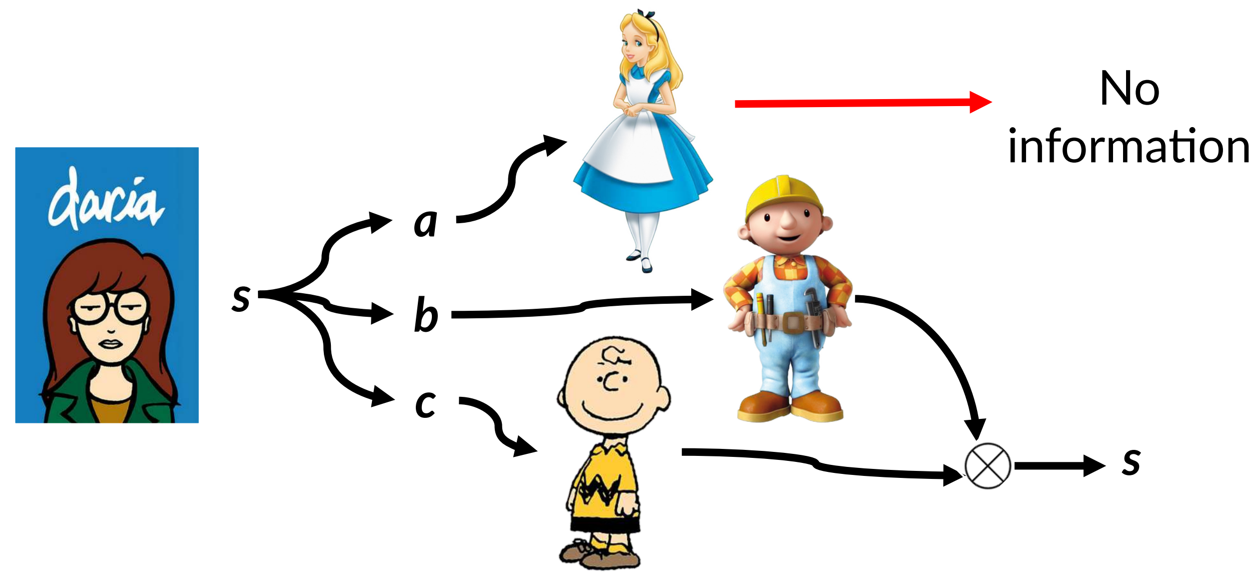
Y. Cai et al, Nat. Comm. 8 (2017)

New theory

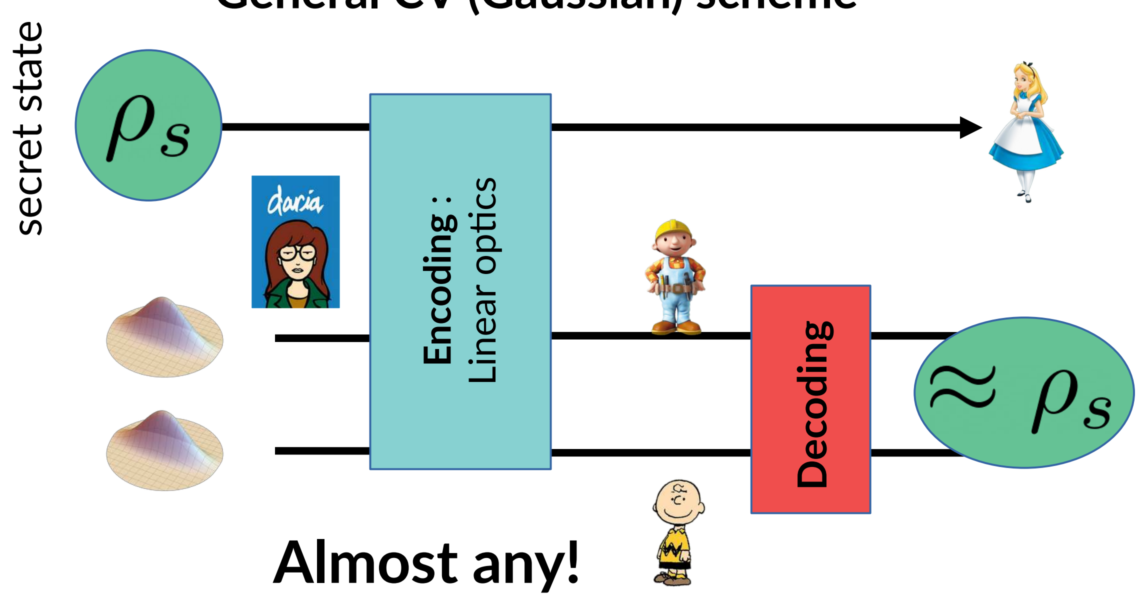
F. Arzani et al, PRA 100 (2019)

Quantum cryptography : Quantum state sharing

Multi-party cryptographic primitive
to securely share a quantum state



General CV (Gaussian) scheme

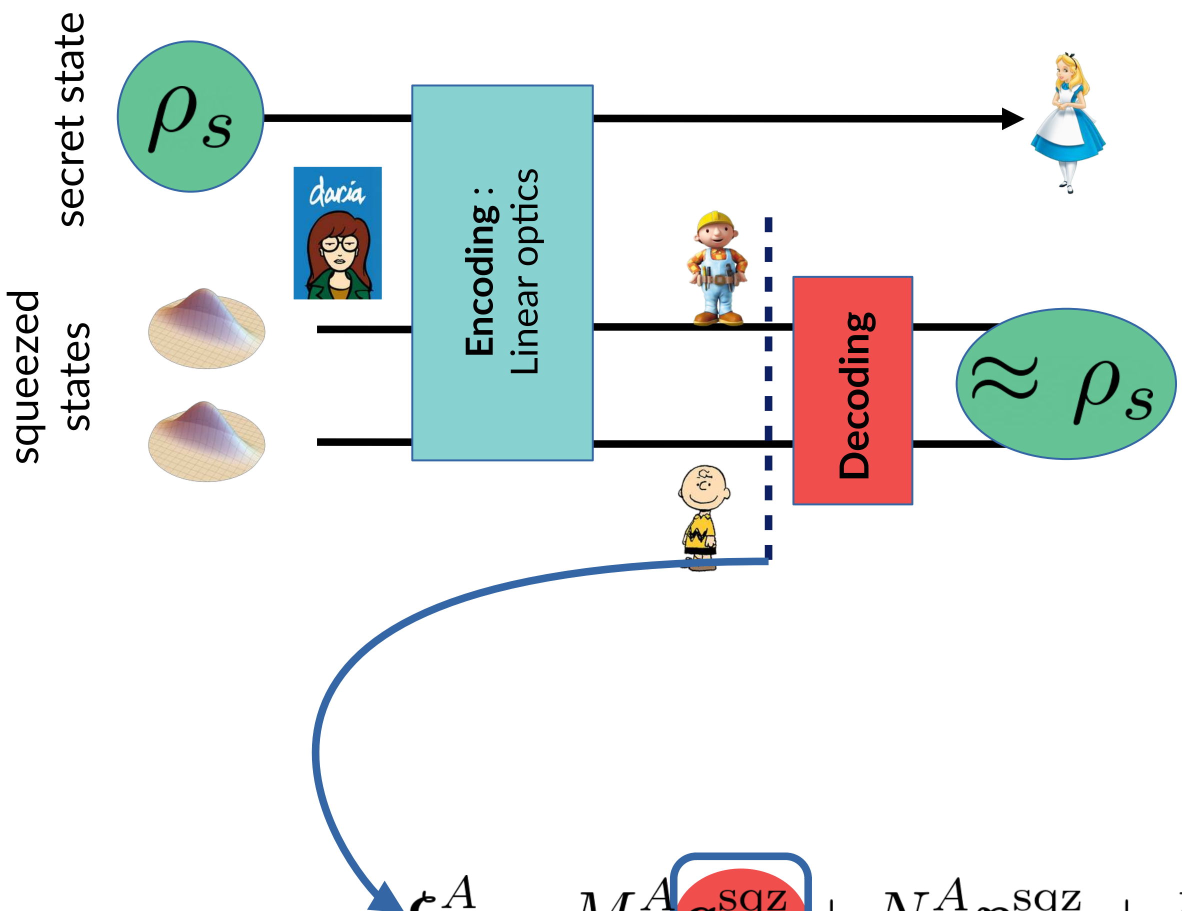


Quantum state sharing with
almost any passive interferometer

- Generalizes previous protocols
- Experimentally friendly
- Related to erasure correcting codes

Y. Cai et al, Nat. Comm. 8 (2017)
F. Arzani et al, PRA 100 (2019)

Random schemes for quantum state sharing



$$\boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{p} \end{pmatrix} \qquad U_G^\dagger \boldsymbol{\xi} U_G = S\boldsymbol{\xi} + \boldsymbol{x}$$

$$\begin{pmatrix} q_j^{\mathrm{sqz}} \\ p_j^{\mathrm{sqz}} \end{pmatrix} = \begin{pmatrix} e^{r_j} q_j^{(0)} \\ e^{-r_j} p_j^{(0)} \end{pmatrix}$$

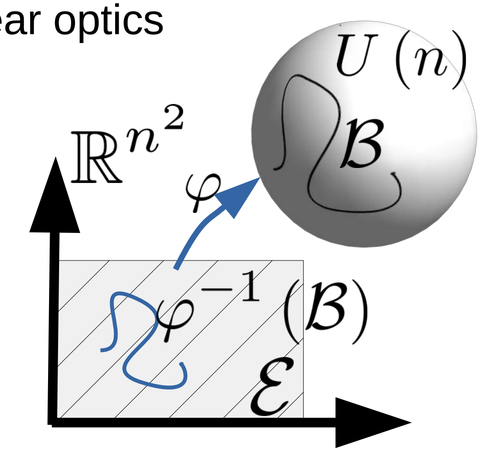
$$\boldsymbol{\xi}^A = M^A \boldsymbol{q}^{\mathrm{sqz}} + N^A \boldsymbol{p}^{\mathrm{sqz}} + H^A \boldsymbol{\xi}^{\mathrm{s}}$$

$A = \{$  ,  $\}$

Goal: 1) Get rid of these

2) solve for these

Linear optics



« Bad » interferometers:

$$\det\left(RH^A\right) = 0$$

Y. Cai et al, Nat. Comm. 8 (2017)
F. Arzani et al, PRA 100 (2019)

Correcting stochastic errors

VOLUME 80, NUMBER 18 PHYSICAL REVIEW LETTERS 4 MAY 1998

Error Correction for Continuous Quantum Variables

Samuel L. Braunstein

SEECS, University of Wales, Bangor LL57 1UT, United Kingdom

(Received 21 November 1997)

We propose an error correction coding algorithm for continuous quantum variables. We use this algorithm to construct a highly efficient 5-wave-packet code which can correct arbitrary single wave-packet errors. We show that this class of continuous variable codes is robust against imprecision in the error syndromes. A potential implementation of the scheme is presented. [S0031-9007(98)05865-7]

Journal of Modern Optics
Vol. 57, No. 19, 10 November 2010, 1965–1971

Taylor & Francis
Taylor & Francis Group

A note on quantum error correction with continuous variables

Peter van Loock*

Optical Quantum Information Theory Group, Max Planck Institute for the Science of Light, Institute of Theoretical Physics I, Universität Erlangen-Nürnberg, Staudtstr. 7/B2, 91058 Erlangen, Germany

(*Received 26 February 2010; final version received 30 May 2010*)

$$W_{\text{out}}(x,p) = (1-\gamma)W_{\text{in}}(x,p) + \gamma W_{\text{error}}(x,p)$$

Same as input *except for a few modes*

We demonstrate that continuous-variable quantum error correction based on Gaussian ancilla states and Gaussian operations (for encoding, syndrome extraction, and recovery) can be very useful to suppress the effect of non-Gaussian error channels. For a certain class of stochastic error models, reminiscent of those typically considered in the qubit case, quantum error correction codes designed for single-channel errors may enhance the transfer fidelities even when errors occur in every channel employed for transmitting the encoded state. In fact, in this case, the error-correcting capability of the continuous-variable scheme turns out to be higher than that of its discrete-variable analogues.

No-Go results for Gaussian operations

Cannot correct Gaussian noise

PRL **102,** 120501 (2009) PHYSICAL REVIEW LETTERS week ending 27 MARCH 2009

No-Go Theorem for Gaussian Quantum Error Correction

Julien Niset,[1] Jaromír Fiurášek,[2] and Nicolas J. Cerf[1,3]

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[3]*Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

(Received 20 November 2008; published 24 March 2009)

We prove that Gaussian operations are of no use for protecting Gaussian states against Gaussian errors in quantum communication protocols. Specifically, we introduce a new quantity characterizing any single-mode Gaussian channel, called *entanglement degradation*, and show that it cannot decrease via Gaussian encoding and decoding operations only. The strength of this no-go theorem is illustrated with some examples of Gaussian channels.

⇒ fully Gaussian schemes are useless for loss, thermal noise, …

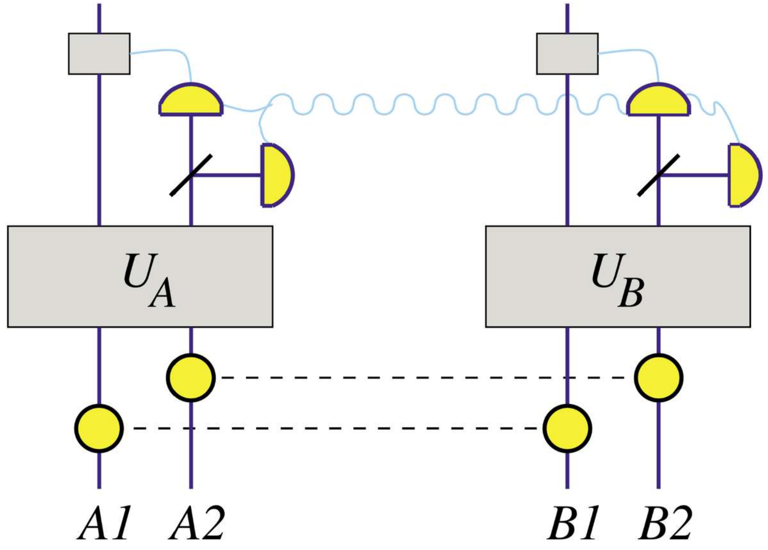
No entanglement distillation

VOLUME 89, NUMBER 13 PHYSICAL REVIEW LETTERS 23 SEPTEMBER 2002

Distilling Gaussian States with Gaussian Operations is Impossible

J. Eisert, S. Scheel, and M. B. Plenio
QOLS, Blackett Laboratory, Imperial College of Science, Technology and Medicine, London, SW7 2BW, United Kingdom
(Received 15 April 2002; published 4 September 2002)

We show that no distillation protocol for Gaussian quantum states exists that relies on (i) arbitrary local unitary operations that preserve the Gaussian character of the state and (ii) homodyne detection together with classical communication and postprocessing by means of local Gaussian unitary operations on two symmetric identically prepared copies. This is in contrast to the finite-dimensional case, where entanglement can be distilled in an iterative protocol using two copies at a time. The ramifications for the distribution of Gaussian states over large distances will be outlined. We also comment on the generality of the approach and sketch the most general form of a Gaussian local operation with classical communication in a bipartite setting.



No computational advantage

VOLUME 88, NUMBER 9 PHYSICAL REVIEW LETTERS 4 MARCH 2002

Efficient Classical Simulation of Continuous Variable Quantum Information Processes

Stephen D. Bartlett and Barry C. Sanders
Department of Physics and Centre for Advanced Computing–Algorithms and Cryptography, Macquarie University, Sydney, New South Wales 2109, Australia

Samuel L. Braunstein and Kae Nemoto
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(Received 11 September 2001; revised manuscript received 26 November 2001; published 14 February 2002)

We obtain sufficient conditions for the efficient simulation of a continuous variable quantum algorithm or process on a classical computer. The resulting theorem is an extension of the Gottesman-Knill theorem to continuous variable quantum information. For a collection of harmonic oscillators, any quantum process that begins with unentangled Gaussian states, performs only transformations generated by Hamiltonians that are quadratic in the canonical operators, and involves only measurements of canonical operators (including finite losses) and suitable operations conditioned on these measurements can be simulated efficiently on a classical computer.

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Positive Wigner Functions Render Classical Simulation of Quantum Computation Efficient

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We show that quantum circuits where the initial state and all the following quantum operations can be represented by positive Wigner functions can be classically efficiently simulated. This is true both for continuous-variable as well as discrete variable systems in odd prime dimensions, two cases which will be treated on entirely the same footing. Noting the fact that Clifford and Gaussian operations preserve the positivity of the Wigner function, our result generalizes the Gottesman-Knill theorem. Our algorithm provides a way of sampling from the output distribution of a computation or a simulation, including the efficient sampling from an approximate output distribution in the case of sampling imperfections for initial states, gates, or measurements. In this sense, this work highlights the role of the positive Wigner function as separating classically efficiently simulable systems from those that are potentially universal for quantum computing and simulation, and it emphasizes the role of negativity of the Wigner function as a computational resource.

⇨ no universal quantum computation

Non-Gaussian resources in optics

Main challenge

Photons interact weakly ⇨ Easy to protect fragile quantum states

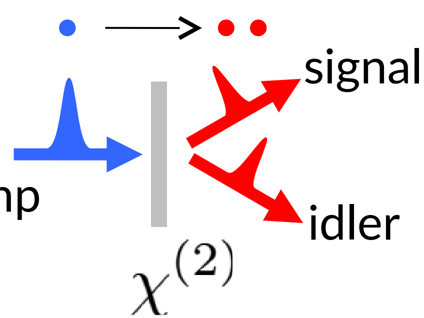
Photons interact weakly ⇨ Very hard to entangle single photons, implement coherent non-Gaussian evolution

Fock states and number resolving detection

$$\hat{a}^\dagger \hat{a}|n\rangle = n|n\rangle$$

$$\mathrm{POVM} : \{|0\rangle\langle 0|,\ |1\rangle\langle 1|,\ |2\rangle\langle 2|,\ \dots\}$$

Heralded Fock states:



1. Prepare

$$|\mathrm{TMSV}\rangle = \frac{1}{\cosh r}\sum_{n=0}^{\infty}(-e^{i\phi}\tanh r)^n|nn\rangle$$

2. Measure $\hat{n}_2$

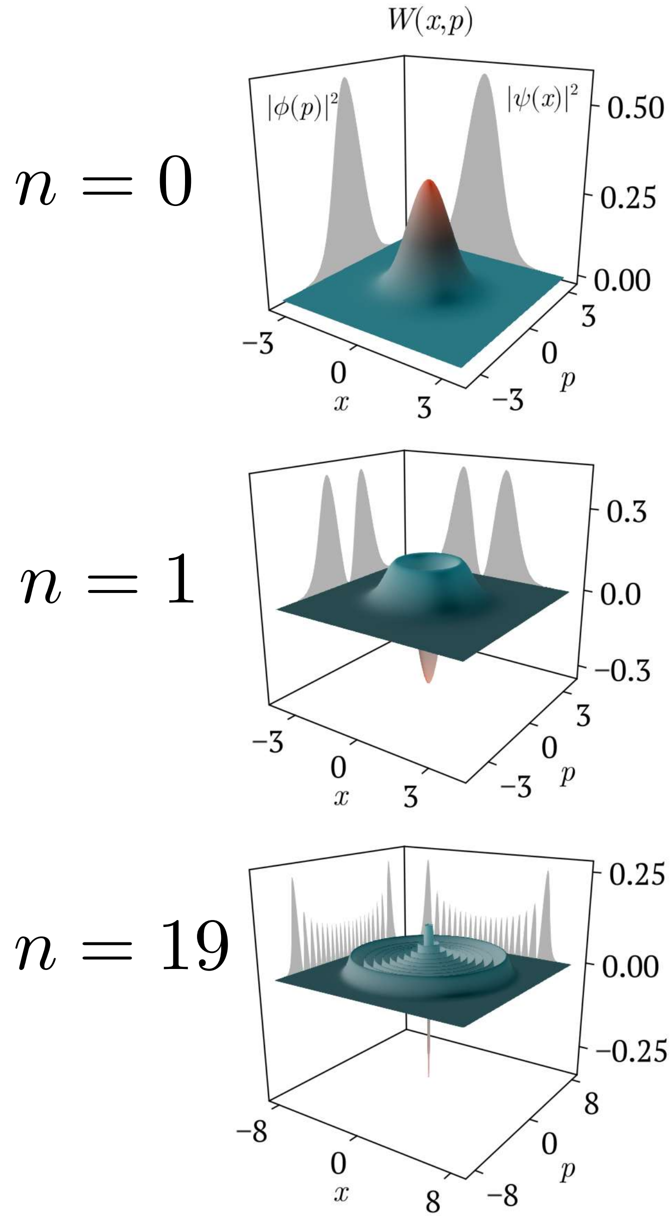
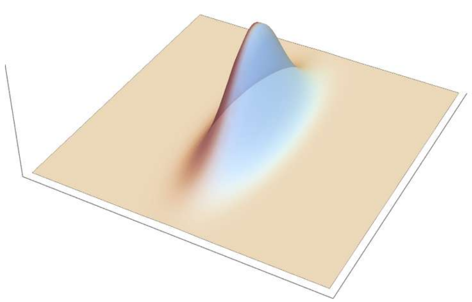
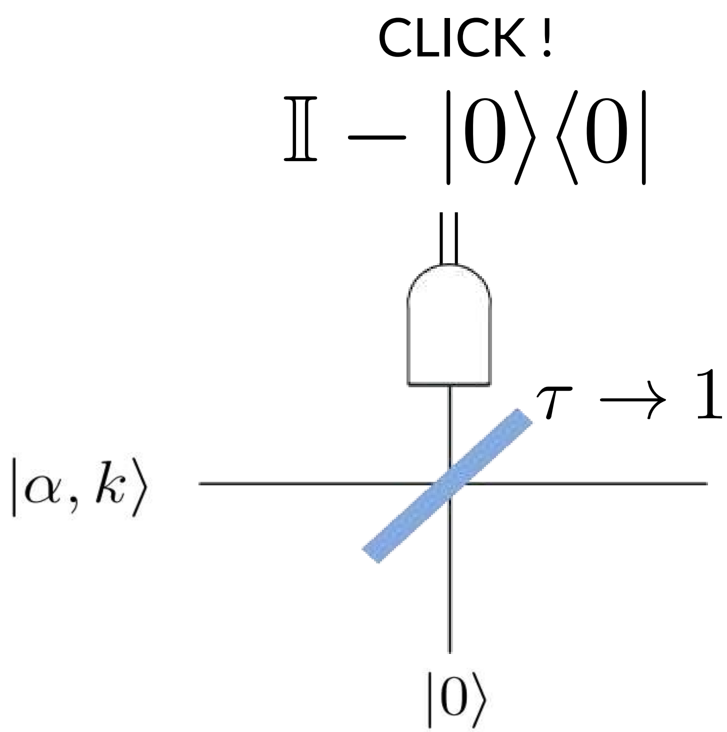


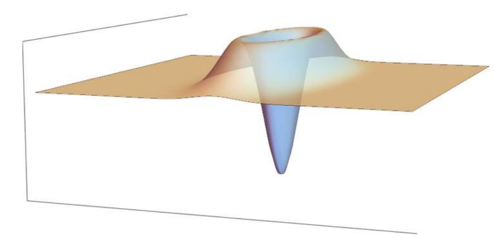
Image : Ashton Bradley, Wikipedia

“on-off” (“click”) detectors

$$\mathrm{POVM}:\{|0\rangle\langle 0|,\ \mathbb{I}-|0\rangle\langle 0|\}$$





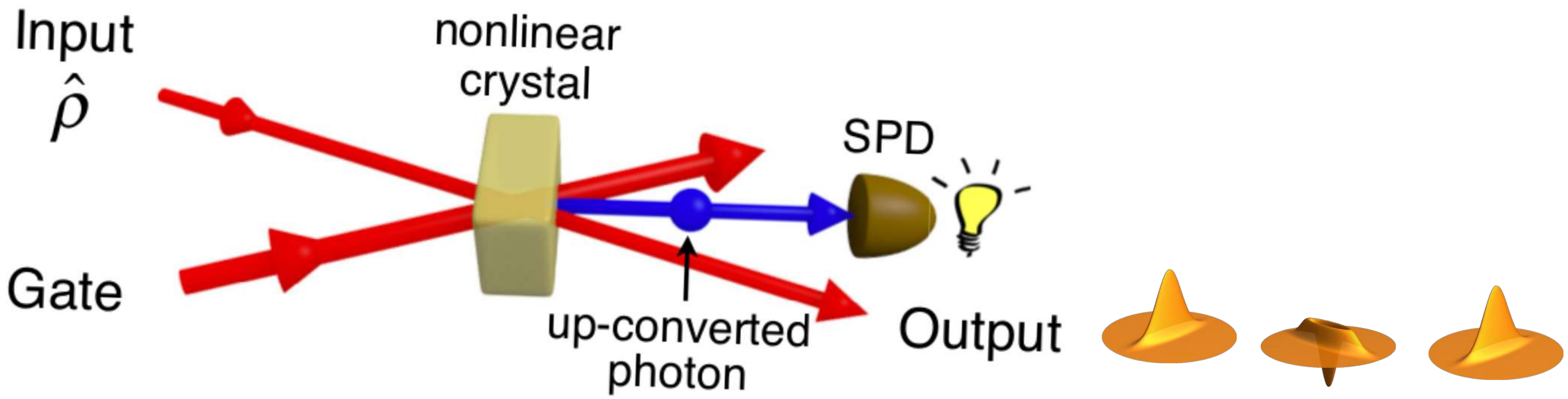


A. Ourjoumtsev et al, Science 312(5770):83–86 (2006)

A. Zavatta et al, NJP 10(12):123006 (2008)

“on-off” (“click”) detectors

$$\mathrm{POVM}: \{|0\rangle\langle 0|,\ \mathbb{I} - |0\rangle\langle 0|\}$$



A. Eckstein et al, Opt. Expr. 19;15 (2011)
Y. S. Ra et al, PRX 7, 031012 (2017)

Protocols using non-Gaussian resources

Quantum computational advantage

Boson sampling:

Classically hard to sample from the output probability distribution

S. Aaronson & A. Archipov, Th. of Computing (2013)
B. T. Gard et al, World Scientific Publishing Co (2015)

Universal CV "computation"

Continuous Variables

qu**modes** $|\psi\rangle \in \left(L^2(\mathbb{R},\mathbb{C})\right)^{\otimes n} \mapsto e^{-itH(\hat{\vec{a}},\hat{\vec{a}}^\dagger)}|\psi\rangle$

General polynomial

Computation with arbitrary encoding

Universal set:

$$\left\{e^{i\hat{q}s}, e^{i\hat{q}^2 s}, e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, e^{i\hat{q}_1\otimes\hat{q}_2}, e^{i\hat{q}^3 s}\right\}$$

Single-mode, Gaussian

Non-Gaussian

Two-modes C_Z

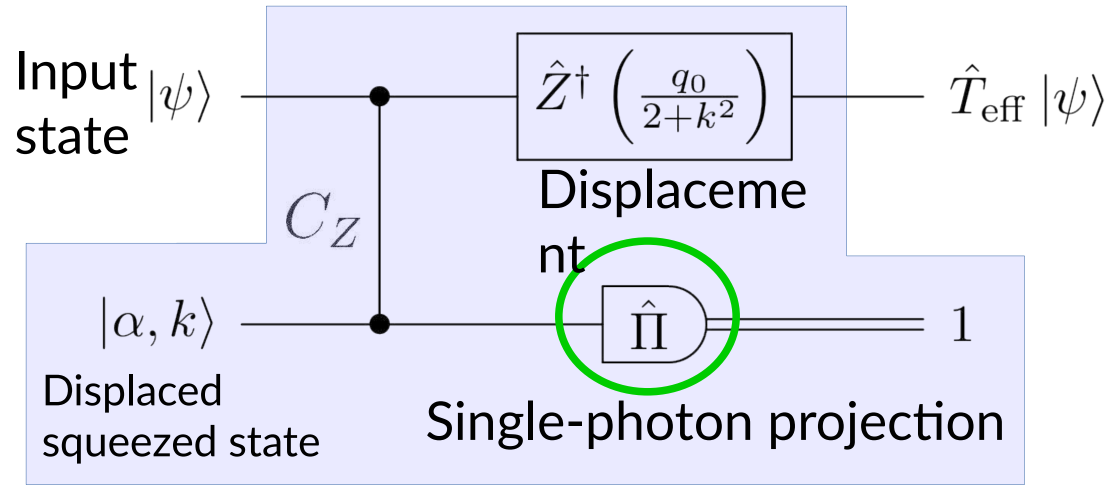
$$e^{itH_2}e^{itH_1}e^{-itH_2}e^{-itH_1} = e^{-t^2[H_2,H_1]} + O(t^3)$$

Can increase degree if at least one is > 2

S. Lloyd, S. L. Braunstein, PRL 82 (1999)

Polynomial approximations to NG gates

Procedure:
1. Entangle input to a Gaussian state
2. Detect a single photon (probabilistic)
3. Perform correction
4. Repeat



Effective transformation

$$\hat{T}_{\text{eff}} = \tilde{\mathcal{N}} \exp\left\{-\left(\frac{k^2}{4+2k^2}\right)(\hat{q}+p_0)^2\right\}\left(\hat{q}-\lambda(\alpha,k)\right)$$

Normalization

Gaussian envelope

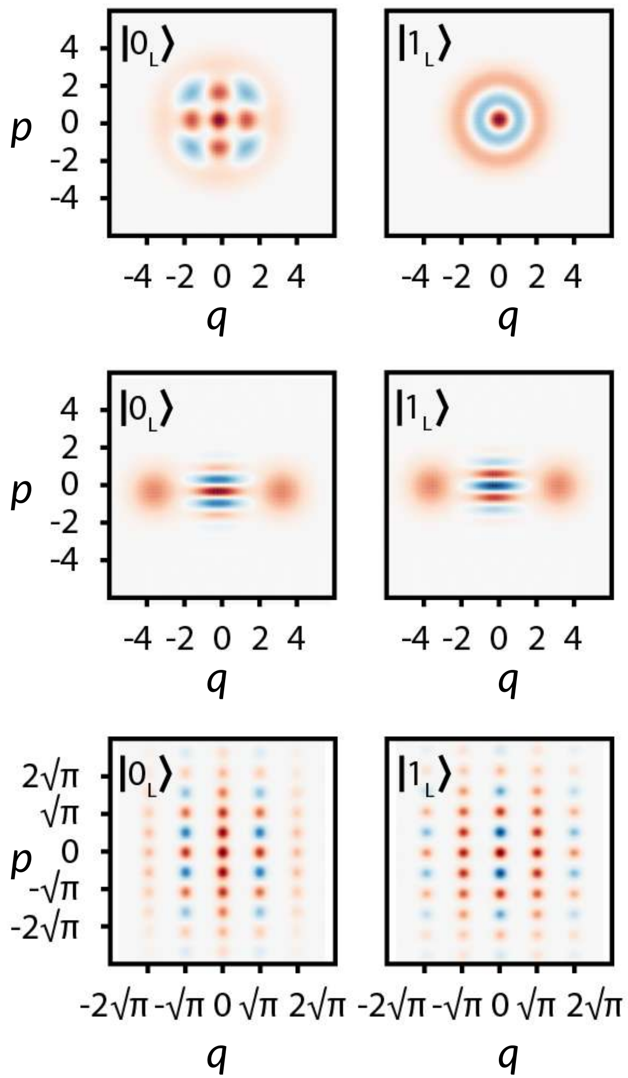
Monomial in *q*

$$e^{i\nu\hat{q}^3} \approx \mathbb{I} + i\nu\hat{q}^3 = (\hat{q}-\lambda_1)(\hat{q}-\lambda_2)(\hat{q}-\lambda_3)$$

Single-photon non-unitary operations can be used to approximate non-Gaussian *unitary* evolution

FA, N. Treps, G. Ferrini, PRA 95 (2017)

Protecting from Gaussian error channels



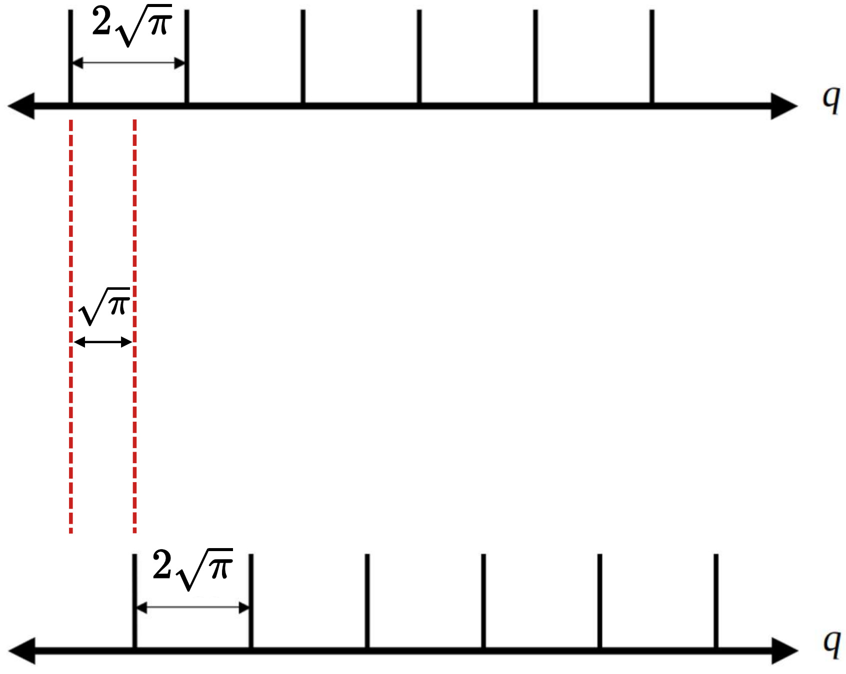
Name:	Designed for:	Structure:
Binomial	photon loss/gain	Superpositions of non-contiguous Fock states
Cat	photon loss/gain	Superpositions of coherent states
GKP	Displacements	Superpositions of position eigenstates

Joshi, Noh, Gao, Q. Sc. Tech. 6 (2021)

Gottesman-Kitaev-Preskill codes I

$$|0_{\mathrm{L}}\rangle = \sum_{k=-\infty}^{\infty} \left|2k\sqrt{\pi}\right\rangle_q = \sum_{k=-\infty}^{\infty} \left|k\sqrt{\pi}\right\rangle_p$$

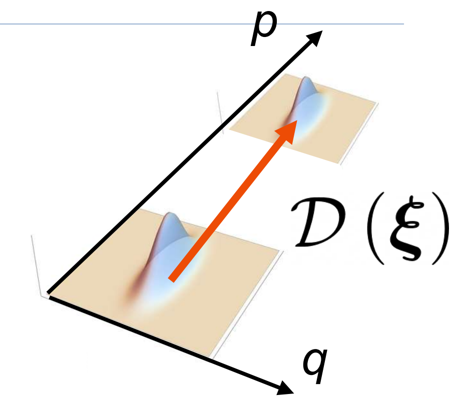
$$|1_{\mathrm{L}}\rangle = \sum_{k=-\infty}^{\infty} \left|(2k+1)\sqrt{\pi}\right\rangle_q = \sum_{k=-\infty}^{\infty} (-1)^k \left|k\sqrt{\pi}\right\rangle_p$$



D. Gottesman, A. Kitaev, J. Preskill PRA 64 (2001)

Gottesman-Kitaev-Preskill codes II

$$\{D(\boldsymbol{\xi}_1), \ldots, D(\boldsymbol{\xi}_{2n})\} \implies \text{Code: } D(\boldsymbol{\xi}_j)|\psi\rangle = |\psi\rangle \quad \forall j$$



$$D(\boldsymbol{\xi}_j)D(\boldsymbol{\xi}_k) = D(\boldsymbol{\xi}_k)D(\boldsymbol{\xi}_j)$$

$$\mathcal{S} = \langle D(\boldsymbol{\xi}_1), \ldots, D(\boldsymbol{\xi}_{2n})\rangle$$

$$D(\boldsymbol{\xi}_j)D(\boldsymbol{\xi}_k) = e^{i\phi_{jk}} D(\boldsymbol{\xi}_j + \boldsymbol{\xi}_k)$$

$$\mathcal{S} \cong \mathcal{L} = \{\textstyle\sum_j z_j \boldsymbol{\xi}_j \ : \ z_j \in \mathbb{Z}\}$$

D. Gottesman, A. Kitaev, J. Preskill PRA 64 (2001)

The lattice point of view

For exponential noise suppression: more oscillators

Up to now: *concatenation* → regard as effective qubits, add qubit-level code
→ "lattice picture" only for individual oscillators,
not for **whole code**

Q: Can lattice properties be exploited more? Yes!

upshot:
lattices are very well studied!

J. Conway and N. Sloane.
Sphere packings, lattices and groups, volume 290. 1988

...but not so much for GKP!

Gottesman, Kitaev, Preskill PRA 64 (2001)
Harrington, Preskill PRA 64 (2001)
Hänggli, Heinze, König, PRA 102 (2020)
Hänggli, König, IEEE TIT 68(2) (2021)

Schmidt, van Loock, PRA 105 (2022)
Royer, Singh, Girvin, PRX Quantum 105 (2022)
Lin, Chamberland, Noh PRX Quantum 4 (2023)
Conrad, Eisert, Seifert, arXiv:2303.02432 (2023)

J. Conrad, J. Eisert, FA, Quantum (2022)

- Code properties from lattice bases
- Symplectic operations
- Distance bounds for GKP codes
- Decoding problem and Θ functions
- GKP codes beyond concatenation

Thank you!