Laboratoire Kastler Brossel
Physique quantique et applications

Polynomial approximation of non-Gaussian unitaries by counting one photon at a time

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ANR erc QCUMbER cnrs dépasser les frontières

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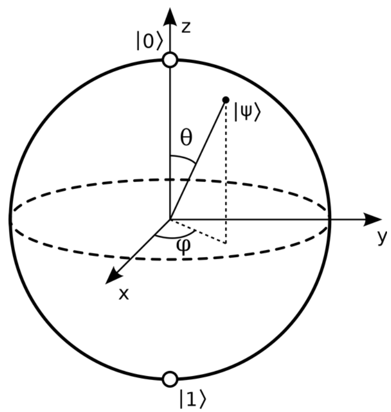
Continuous-variable quantum information

DV : information encoded in qu-bits



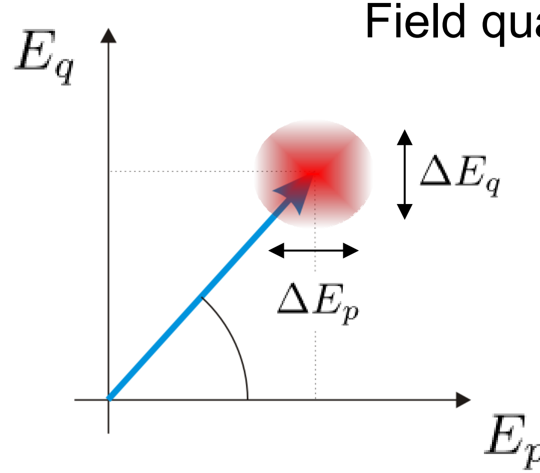
Optical ex:

Polarization of single photon

CV : information encoded in observables with continuous spectrum, e.g. : $\hat{q}$, $\hat{p}$

Optical ex:

Field quadratures



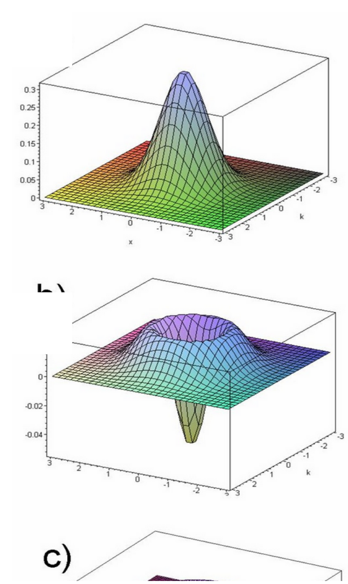
$$\hat{E}_Q \propto \hat{a} + \hat{a}^\dagger$$

$$\hat{E}_P \propto i(\hat{a}^\dagger - \hat{a})$$

Wigner function

$$W\left(q,p\right) = \frac{1}{2\pi}\int \mathrm{d}x e^{ipx}\left\langle q - \frac{x}{2}\right|_q \hat{\rho}\left|q + \frac{x}{2}\right\rangle_q$$

$$\left\langle \hat{O}\right\rangle = \mathrm{Tr}\left[\hat{\rho}\hat{O}\right] = \int \mathrm{d}q\mathrm{d}p W_\rho\left(q,p\right) W_O\left(q,p\right)$$



For **pure states**:

Gaussian
iff
Non-negative

R. Hudson
Rep. Math. Phys 6, 249 (1974)

(Quantum) Computing and continuous variables

Discrete encoding $\vec{x} = 100111101010101001\ldots$

bits $\vec{x} \mapsto b(\vec{x})$ For any boolean function

qubits $|\vec{x}\rangle|\phi\rangle \mapsto U_b|\vec{x}\rangle|\phi\rangle = |\vec{x}\rangle|b(\vec{x})\rangle$

Continuous Variables

qu**modes** $|\psi\rangle \in \left(L^2(\mathbb{R}, \mathbb{C})\right)^{\otimes n} \mapsto e^{-itH(\hat{\vec{a}}, \hat{\vec{a}}^\dagger)}|\psi\rangle$

General polynomial

Computation with arbitrary encoding

Universal set:

$$\left\{\underbrace{e^{i\hat{q}s}, e^{i\hat{q}^2 s}, e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}}, e^{i\hat{q}_1\otimes\hat{q}_2}, e^{i\hat{q}^3 s}\right\}$$

Single-mode, Gaussian

Two-modes C_Z

Non-Gaussian

S. Lloyd and S. L. Braunstein PRL 82, 1784 (1999)

(Quantum) Computing and continuous variables

No quantum advantage without non-Gaussianity !

A. Mari and J. Eisert
PRL 109, 230503 (2012)

Actually, it's negativity of the WF...

Continuous Variables

qu**modes** $|\psi\rangle \in \left(L^2(\mathbb{R},\mathbb{C})\right)^{\otimes^n} \mapsto e^{-itH\left(\hat{\vec{a}},\hat{\vec{a}}^\dagger\right)}|\psi\rangle$

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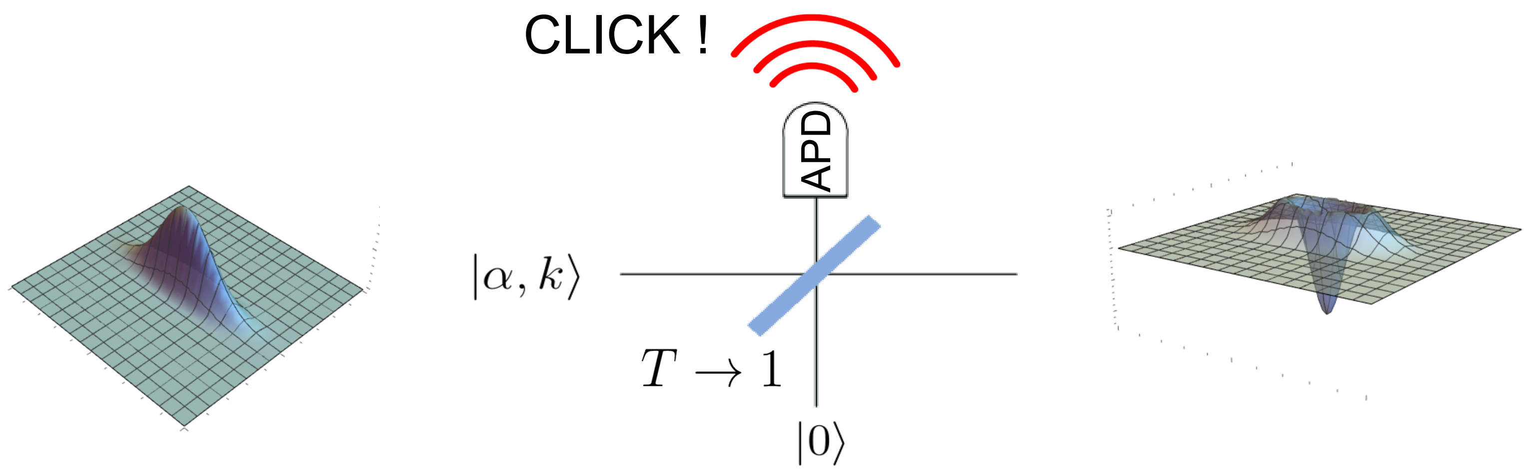
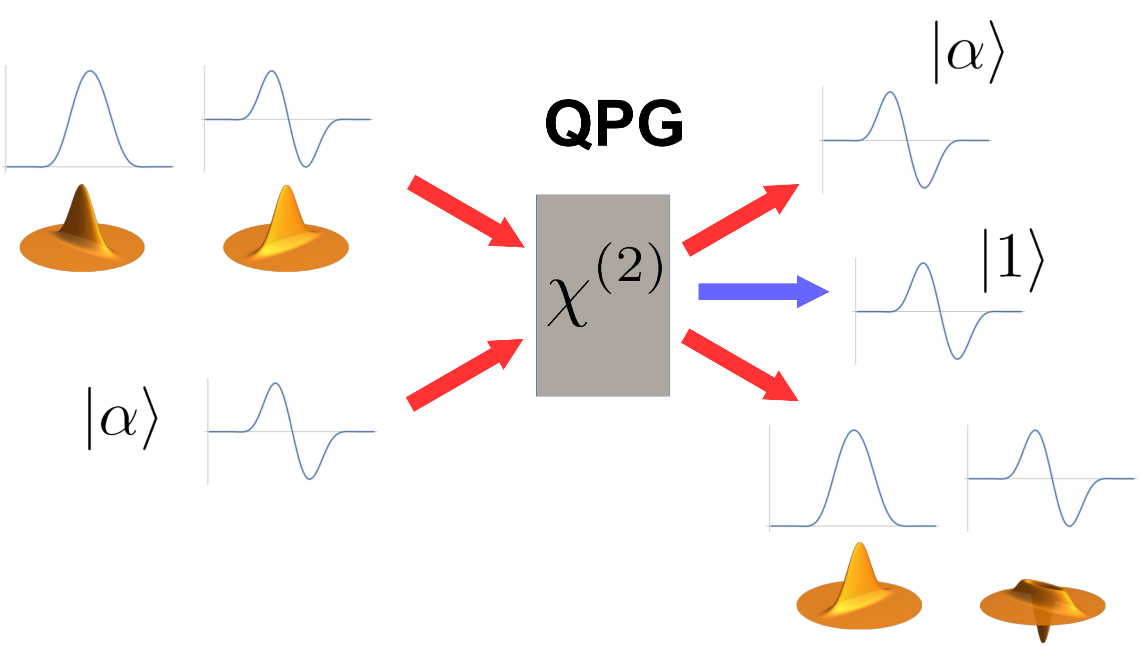
Introducing Negativity: Photon Subtraction



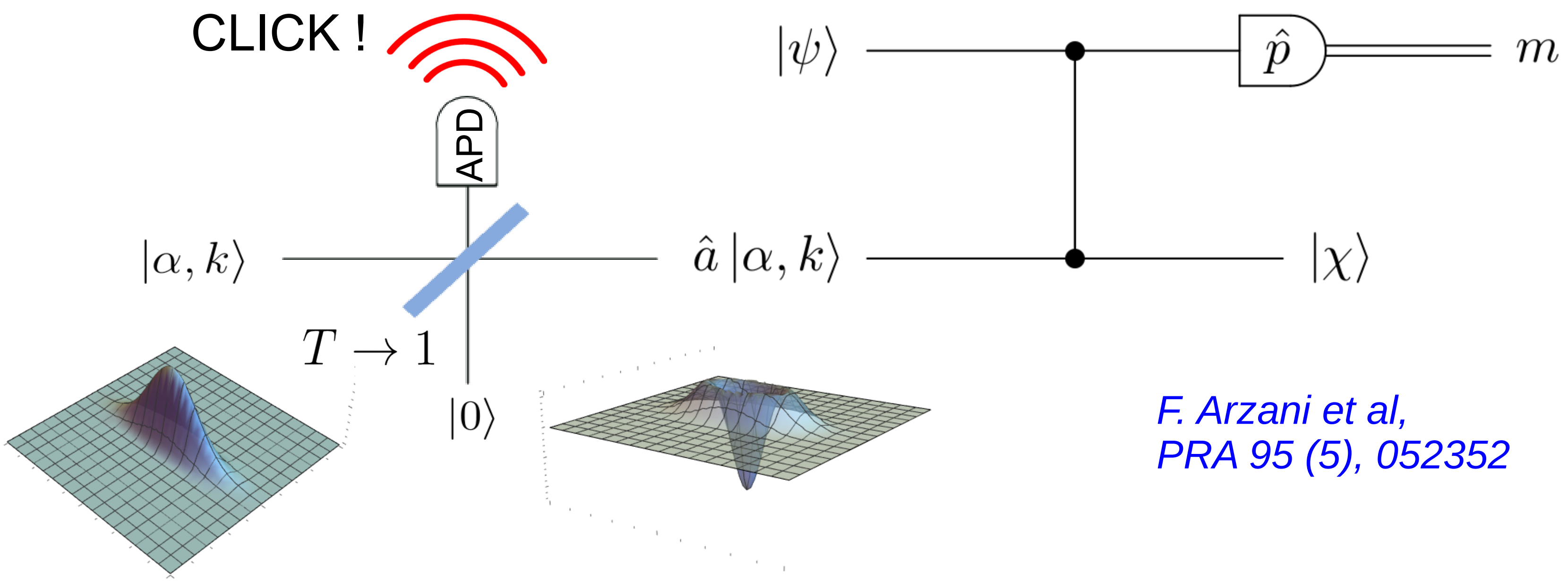
Motivation: experiments in LKB!



A. Eckstein et al,
Opt. Expr. 19;15 (2011)

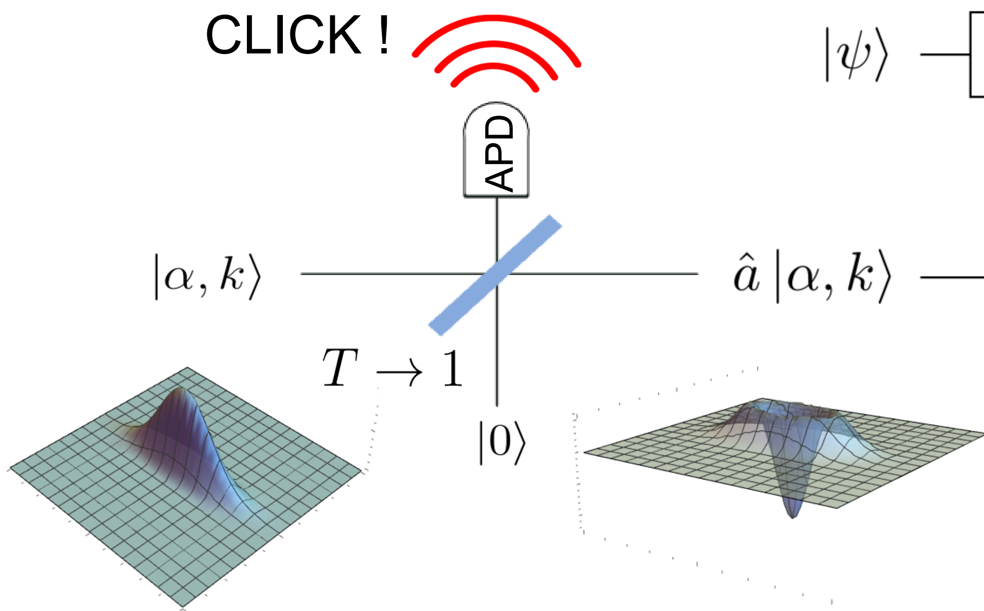
J. S. Ra et al,
PRX 7, 031012 (2017)

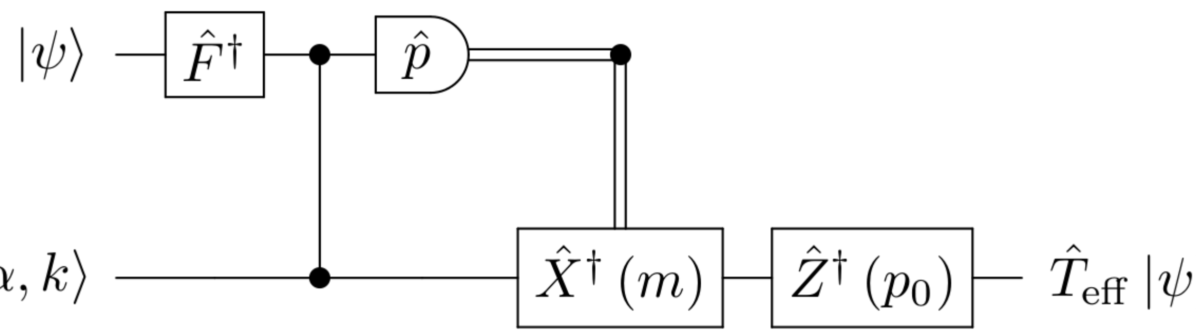
Measurement-based approach



F. Arzani et al,
PRA 95 (5), 052352

Cosmetics





F. Arzani et al,
PRA 95 (5), 052352

$$\hat{T}_{\text{eff}} = \mathcal{N} \exp\left\{ -\frac{(\hat{q} - q_0 + m)^2}{k^2} \right\} \left(\hat{q} - \lambda(\alpha, k, m) \right)$$

Normalization — envelope — Monomial in *q*

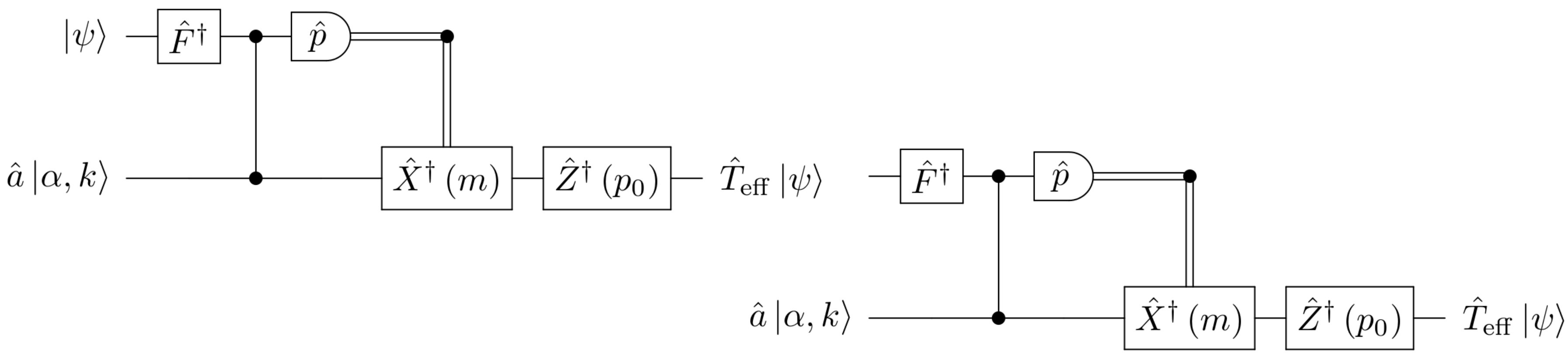
$$\lambda(\alpha, k, m) = -\left(\frac{2}{k^2 - 2}\right) q_0 - i \left(\frac{k^2}{k^2 - 2}\right) p_0 - m$$

<u>Depends on</u>

- Experimental parameters
- Measurement (!!!)

Approximating Unitary operators

Repeated application: **polynomial** in the quadratures



Three times: get something like

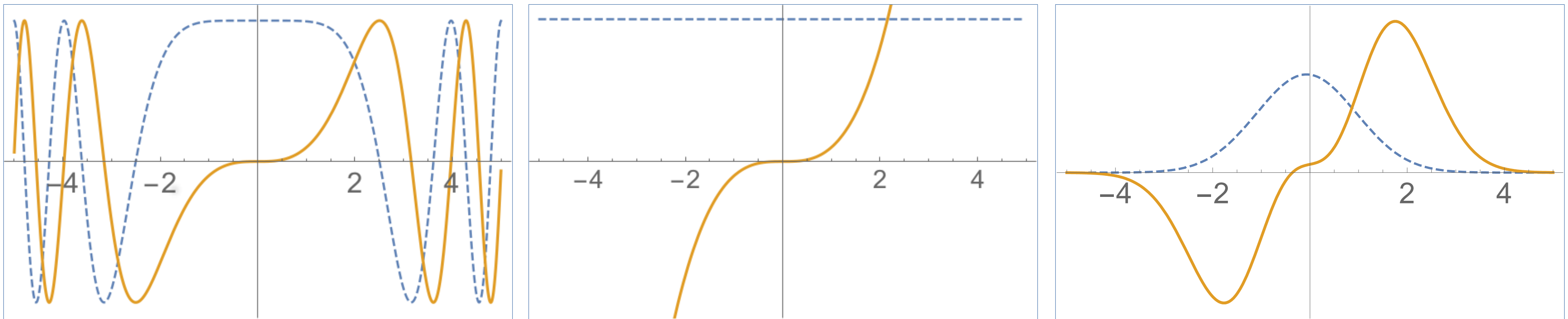
$$e^{i\nu\hat{q}^3} \approx \mathbb{I} + i\nu\hat{q}^3 = (\hat{q} - \lambda_1)\,(\hat{q} - \lambda_2)\,(\hat{q} - \lambda_3)$$

P. Marek et al
PRA 84(5), 053802 (2011)

K. Marshall et al
PRA 91, 032321 (2015)

Approximating Unitary operators

$e^{0.1ix^3}$ $\quad$ $1+0.1ix^3$



$$\mathcal{T}_{\text{eff}}\left(x,\vec{m}\right)\propto\prod_{i=1}^{3}\left[\mathcal{G}\left(x,\alpha_i,k,m_i\right)\left(x-\lambda\left(\alpha_i,k,m_i\right)\right)\right]$$

Finite **success probability**: acceptance region Ω

≈ 10^{-9} - 10^{-12}

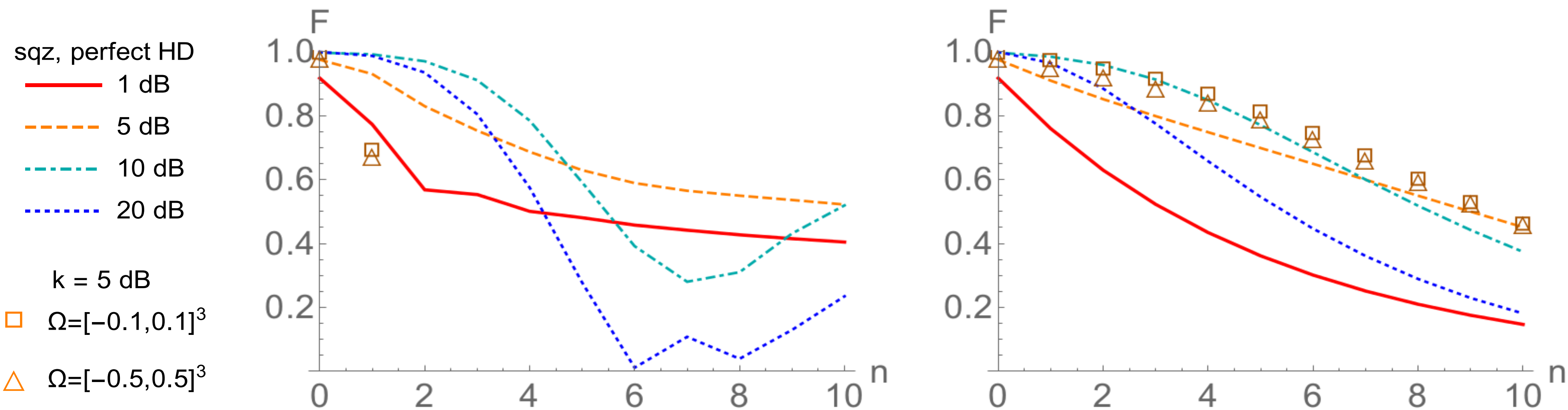
Average output state

$$\rho_\Omega=\int_\Omega \mathrm{d}^n m\frac{p\left(\vec{m}\right)}{p_\Omega}\hat{\mathcal{T}}_{\text{eff}}\left(\vec{m}\right)\left|\psi\right\rangle\left\langle\psi\right|\hat{\mathcal{T}}_{\text{eff}}^{\dagger}\left(\vec{m}\right)$$

Benchmarking: Fidelity of the efficient gate

$$\rho_\Omega = \int_\Omega \mathrm{d}^n m \frac{p(\vec{m})}{p_\Omega} \hat{\mathcal{T}}_{\text{eff}}(\vec{m}) |\psi\rangle \langle\psi| \hat{\mathcal{T}}^\dagger_{\text{eff}}(\vec{m})$$

$$\mathcal{F} = \sqrt{\langle\psi| \hat{U}^\dagger \rho \hat{U} |\psi\rangle} \qquad \hat{U} = e^{i\nu\hat{q}^3}$$



Input: Fock states Coherent states

State Preparation

Know the input: increase success probability

$$\left(\hat{q}-\lambda\left(\alpha,k,m\right)\right) \rightsquigarrow \lambda\left(\alpha,k,m\right) = -\left(\frac{2}{k^2-2}\right)q_0 - i\left(\frac{k^2}{k^2-2}\right)p_0 - m$$

Ex: Cubic phase state

$$\left|\gamma\left(\nu\right)\right\rangle = \hat{\gamma}\left(\nu\right)\left|0\right\rangle_p \to \hat{\gamma}_{\text{appr}}\left(\nu\right)\left|k_{in}\right\rangle_p$$

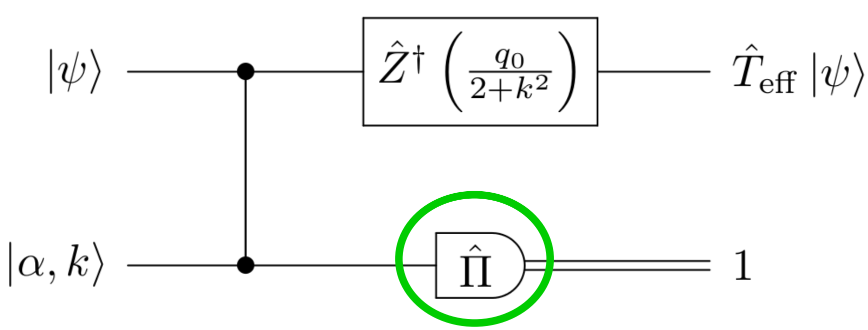
Fidelity = 0.9 with

$$\hat{\gamma}\left(\nu\right)\left|k_{in}\right\rangle_p = e^{i\nu\hat{q}^3}\left|k_{in}\right\rangle_p$$

for k = 5 dB, k_{in} = 5 dB, v = 0.1

Success probability ~ 10^{-4}

Alternative Scheme: Single-Photon Counter



Projects on **single photon**

Still post-selection but no binning of HD outcomes

~ Conjugate process of

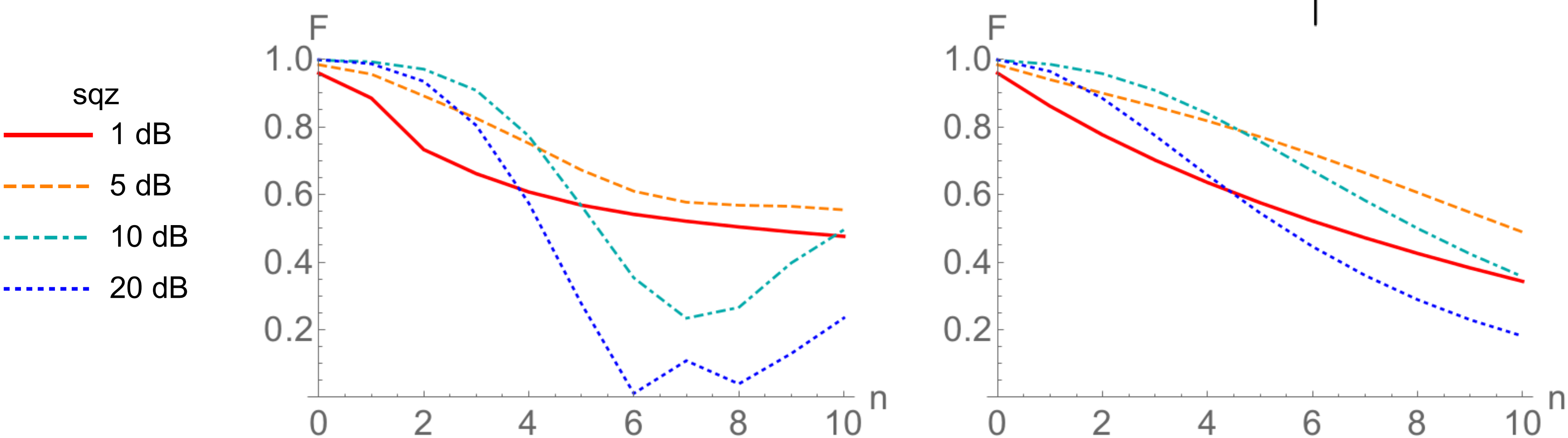
K. Park et al
PRA 90, 013804 (2014)

$$\hat{T}_{\text{eff}} = \tilde{\mathcal{N}} \exp\left\{-\left(\frac{k^2}{4+2k^2}\right)(\hat{q}+p_0)^2\right\}\left(\hat{q}-\lambda(\alpha,k)\right)$$

$$\lambda(\alpha,k) = \frac{2i}{k^2}q_0 - p_0$$

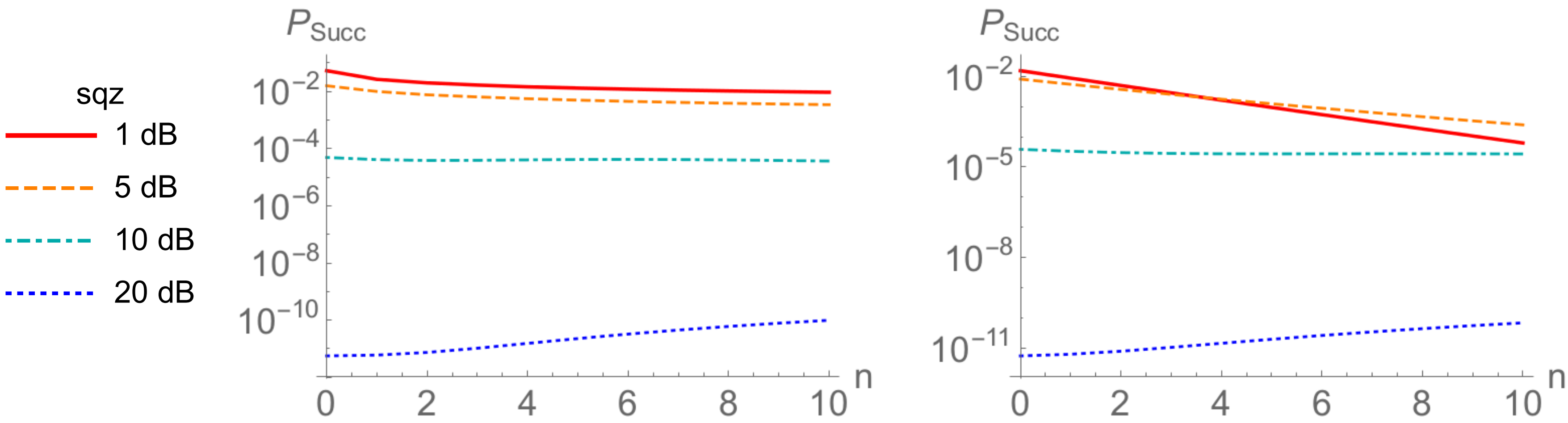
$$\hat{U} = e^{i\nu\hat{q}^3}$$

$$\mathcal{F} = \left|\langle\psi|\,\hat{U}^\dagger\hat{\mathcal{T}}|\psi\rangle\right|$$



Input: Fock states Coherent states

Success Probability



Input: Fock states Coherent states

- Probability of detecting exactly one photon in each step
- Lower for high squeezing: many photons in the ancilla

Summary

- CV quantum computing
- Non-gaussianity through photon-subtraction
- Polynomial approximation of unitaries
- Alternative scheme : single photon counter

Thank you !